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On the peer-to-peer instruction in high school mathematics education, a case study*

Abstract. The purpose of this note is to report on an educational experiment conducted in the academic year 2020/21 in a Polish high school under the pandemic situation. We discuss some various scenarios of implementation of the peer instruction method as developed by Eric Mazur blended with the flipped class approach. The material used during the experiment is discussed in grade 11 and concerns polynomials and their properties. The experiment was conducted in two classes. We were primarily interested in using peer instruction to eliminate errors in students' reasoning resulting from deep misconceptions.

1. Introduction

One of purposes of research on mathematical education is to make teaching and learning mathematics more effective, where effectiveness is measured not only by students scores on exams but also by their attitude to the subject. A lot of attention is devoted to understanding reasoning of students which lead to false answers. Problems with effectiveness of teaching are not restricted to mathematics and they concern all levels of education, see e.g. (Swan, 2023). In the beginning of the 90s Eric Mazur (Mazur, 1997), professor of physics at the Harvard University introduced a new strategy of approaching the effectiveness issues in teaching. This strategy is nowadays known as the *peer instruction* (Crouch, Mazur, 2001).

This article is motivated by these pioneering ideas. One of pillars of the peer instruction is to confront students with questions where a correct answer is not immediately obvious, so that the diversity of answers, revealed on the spot by a clever usage of technology, leads to discussions in the class. Our idea was to modify Mazur's method by including in the set of possible answers such answers which, in our opinion, were far away from the correct ones. Our expectation was

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that such answers should not be even considered as feasible by average students. Surprisingly, this was not the case and we focused on this phenomena. Thus contrary to most experiments of this kind, see e.g. (Tamer, Nurtaç, 2019), we were not interested in leading the students directly to correct answers but rather in the elimination of their misconceptions and reduction/elimination of the most erroneous among students' answers.

In this article we describe four variations of the peer instruction method and present our findings. The experiments were conducted in the academic year 2020/21 in a high school in Poland under the unusual conditions of the COVID-19 pandemic. Whereas distant learning of that period imposed severe problems on the education, we found it very helpful for this project that students were online all the time and thus could take part in quizzes without any additional effort and with no technological barriers. Our work constitutes mainly a didactical proposal supplemented by some findings which might be useful in a more theoretically inclined research work. We hope to come back to this side of our project in the future, when more data is available.

2. Theoretical background

In the early 1990s at Harvard University, professor of experimental physics Eric Mazur noticed the low effectiveness of traditional teaching and ineffective communication between students and the lecturer. Mazur viewed (and still does) the education as a social process which requires good communication on the line teacher - students and maybe even more importantly among the students themselves. This is in full accordance with our view of mathematics (indeed any subject) learning as a social process rather than an individual cognitive experience, see e.g. (Salomon, Perkins, 1998). Mazur's study of this important problem culminated in the 1997 book (Mazur, 1997). Physicist introduced a new method for engaging students in the learning process. The classes were divided into 10+5 minutes segments. For 10 minutes, the students were taught in the lecture-based style some new material and 5 minutes were spent on quick tests for students. After a short test the teacher had information about how well the students acquired the new knowledge. If the students scored in the quiz on the average lower than 90%, then the discussed material had to be repeated. When the result exceeded 90%, the instructor could assume that the material laid out was gained at a level enabling further progress of students. After each quiz, the students consulted their answers in groups - this possibility allowed for the exchange of arguments between the students. During that time, the lecturer listened to the work in groups and gained experience. In this way, teaching became more dynamic, which means that it became more engaging for the students, which in turn resulted in the increase of the effectiveness of the whole process.

In the course Eric Mazur proposed some variations, extensions and modifications to the general outline of the method, which could be applied depending on a particular group of students at hand. The method has been adopted to other branches of science and to other levels of education: from high through the middle to primary schools, see for example (Miller, Santana-Vega, Terrell, 2006)

(calculus), (Crouch, Mazur, 2001; Kola, 2017) (sciences general), (Tullis, Goldstone, 2020) (university physics), (Šestáková, 2016) (highschool mathematics), (Pavlin, Čampa, 2021) (primary school).

Jana Šestáková (Šestáková, 2016) used the peer instruction method in high school (students ages 17-18 years) and observed that consultations prove useful if query voting results of the group of students oscillate between 30% and 70%. She considers the peer discussion in this range as the most appropriate and useful. Additionally, she paid a lot of attention to whether the students change their originally expressed opinion during the discussions. Her study shows that if the correctness of the first voting is below 30%, then additional teacher explanation is in place. On the other hand, if the correctness is above 70%, then it is more effective to pass to the next problem, rather than dwell on the problem grasped by the vast majority of students. She points out that this majority is very likely to convince the small minority without really explaining their reasoning, thus the cognitive effect of such a discussion is very low, if any.

There is a rather big discrepancy between Mazur's threshold of success which is set at 90% and Šestáková's score as low as 70% which made us wonder about results in our experiment. Our findings are presented in the next section and they show in particular that Mazur's threshold is unrealistic in a school class, which consists of people for whom mathematics is not of primary, and probably not even of the secondary interest, contrary to students of physics focused on the subject at one of the world best universities.

On the other hand, the technical difference in Mazur's and Šestáková's approach: Mazur in his experiment used an application collecting data on student returns, whereas Šestáková collected data with the help of flashcards, seems not to affect the results.

3. Methodology

Here we provide information about participants of the experiment, material used and procedures applied.

The group participating in the experiment consisted of 61 students 15–16 years old coming from two different classes at the same level in the same high school. They did not have to participate in the quizzes. In every case it was their individual decision whether or not to express their opinion. This is why the number of votes varies in each question. Student discussions, which are at the core of the method, were carried out in groups of 4–6 students. Similarly to Mazur, we used the pingo mobile application (pingo.coactum.de, 2023) to collect students' votes.

The material covered (see Table 1) belongs to standard curriculum, the teaching units were part of the normal teaching procedures.

The experiment lasted 6 weeks. The observations were divided into four stages. Each stage is characterised by applying a different variant of the peer instruction strategy. The aim of this experiment was to find out which method makes it easier for students to identify answers containing the most factual errors and avoid them. Even though correcting misconceptions is widely discussed in the literature, see e.g. (Bangert-Drowns, Kulik, Kulik, 1991), this perspective seems new in the peer

instruction set-up as most studies focus on identifying the correct answers, without analysing and differentiating between incorrect answers, compare e.g. (Tullis, Goldstone, 2020).

The classes were devoted to the chapter "Polynomials", which is scheduled for 4–6 weeks in the standard curriculum. More specifically we covered the following topics: short multiplication formulas, factorization of polynomials, polynomial division and Bezout's theorem. The coupling of peer instruction variants and the topics is presented in Table 1.

Variant	Topic
1	Short multiplication formulas
2	Factorization of polynomial
3	Polynomials division
4	Bezout's theorem

Table 1: Topics discussed in classes

Coming up with questions not only testing students' performance but also opening door to meaningful exchange of their ideas and ways of reasoning is not an easy task. It is a challenge for teachers who want to apply variants 3 or 4. We present here a sample pair of questions we used in Variant 4, whose topic revolves around Bezout's theorem. Students were instructed at the beginning of the experiment that multiple correct answers are possible.

Sample Question 1. *Consider all polynomials $f(x)$ of degree 2 with $f(-1) = f(1) = 0$. Among these polynomials there are such that the equation*

$$f(x) = 1$$

has: zero, one, two, three solutions.

The most erroneous answer here is "three". After students' discussion, in the next round we tested their comprehension with the following modification.

Sample Question 2. *Consider all polynomials $f(x)$ of degree 3 with $f(-1) = f(0) = f(1) = 0$. Among these polynomials there are such that the equation*

$$f(x) = 1$$

has: zero, one, two, three solutions.

Here the most erroneous answer is "zero". These two questions sparked a great amount of discussion among students.

The variants were implemented in the order indicated in Table 1. The topics were increasingly more complex paralleling the complexity of applied variants. On the other hand students become more and more familiar with various elements of our method and were not distracted from the learning process by technical issues.

4. Peer instruction variants in our experiment

Here we describe main components of each variant of the peer instruction framework, which we applied in the class room. The variants are ordered from the most simple to the most complicated (and time-consuming) one. This ordering agrees with the order they were introduced in our lessons.

4.1. Variant 1 (basic)

In the first variant the idea is to go over the given topic in three cycles, each of them consisting of two parts: a lecture followed by a quiz, see Figure 1. Initially,

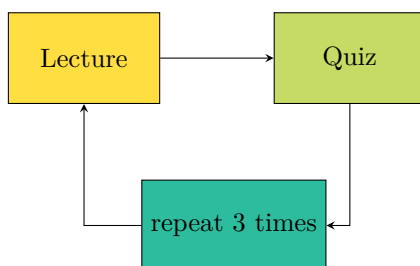


Figure 1: Variant 1

approximately 15 minutes were planned for the first part and 5 minutes for the second one. The first part is the teacher's discussion of a short piece of material. It means, the teacher introduces definitions, theorems, formulas and thoroughly discusses examples. Students solve problems related to the topic and have an opportunity to ask questions. During the second part, students take a short two-questions quiz (2 minutes for each question) to check how well they understand the subject under discussion. This procedure is repeated three times, which means that students answer six questions in total. At each subsequent step the teacher moves on to the next batch of material and extends the previously discussed topic.

The benefit of using this method, besides its simplicity, is that the students feel compelled to pay attention as their knowledge is verified on a regular basis. Also, the effects of this method were very satisfying, with each subsequent question the number of votes for answers with the most serious errors decreasing. However, we must admit that the classroom material to which this method was applied was of introductory level. Moreover, a significant drawback of this method is the inability to measure the real effect of its application as there is no initial test carried out prior to the instructional part to compare with.

4.2. Variant 2 (including flipped class elements)

Recall first that a flipped classroom is a learning concept motivated by the observation that lecture or direct instruction is not the optimal use of time teacher and students spend together in a classroom, see e.g. (Bishop, Verleger, 2013). Instead students encounter information before class, freeing class time for activities

that involve higher order thinking. This approach has to be applied with caution, as too much pre-class activity might endanger the social experience of learning.

Following the idea described above, in the second variant the teacher hands out the teaching material to the students for a self-study at home. Handouts include definitions, theorems, formulas and examples along with detailed reasoning. The classroom lesson begins with an about fifteen-minutes quiz consisting of six questions checking results of the out of the class learning. The next step is to follow the procedure from the first method in the same time frame, which is to go over the topic previously presented in the handed out materials in three cycles, each of which consists of the teacher's explanations and followed by a brief two-question quiz, see Figure 2.

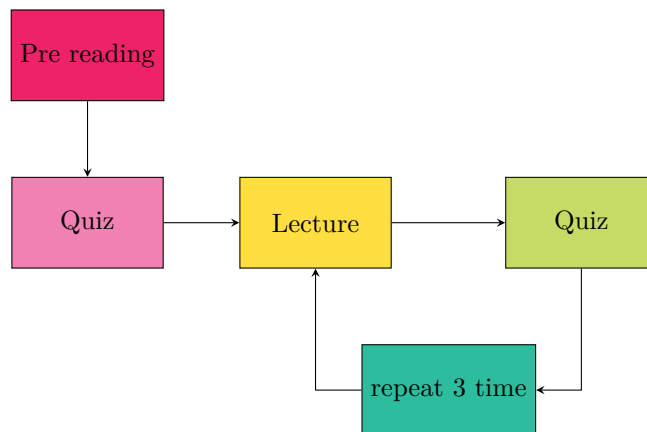


Figure 2: Variant 2

Both the teacher and the students benefit from using this method. Knowing the results of the entrance test the teacher knows which parts of the new material are well assimilated by the students and where the most common mistakes appear. This makes it possible to adjust the course of the lesson on an ongoing basis, for example to use time in the classroom to explore more difficult topics in a greater depth, to make more examples or to focus on the items that caused most trouble to the students. On the other hand, students are already familiar with the new topics (as long as they actually did the homework). Knowing questions from the entrance quiz, they are able to focus their attention and better pick up important points in the teacher's subsequent explanations. An important advantage of this variant is the possibility of referring to the results obtained on the same research group and in the analysis of the same material. Initially, students may feel burdened with this way of working, but at the long run they appreciate the benefits deriving from it: self-reliance in information retrieval, work with the manual and a habit of self-education, which generally gives a good preparation for starting higher level education.

4.3. Variant 3 (classic, including peer discussion)

In the third variant we begin as in the Variant 1. An important difference is that after each quiz the students discuss in groups their answers and are asked to work on the same quiz again. Initially, approximately 15 minutes were planned for the first part and 5 minutes for the second part. Students were informed that there are 5 minutes assigned for their consultations and their re-voting (2 minutes for each question). The students have the opportunity to exchange their arguments and explain their paths of reasoning in groups of 4-6 people (we took the advantage of teaching online and used escape rooms for the purpose of these discussions). Students re-vote after the discussion and then the teacher provides a short analysis of possible reasons for the initial wrong answers. This time planning turned out unrealistic, students discussions took in the mean twice as long as we initially expected. We repeated the scheme three times, which means that students answered six questions during lesson. At each subsequent step the teacher moved on to the next batch of material, extend the previously discussed topic and posed new questions, see Figure 3.

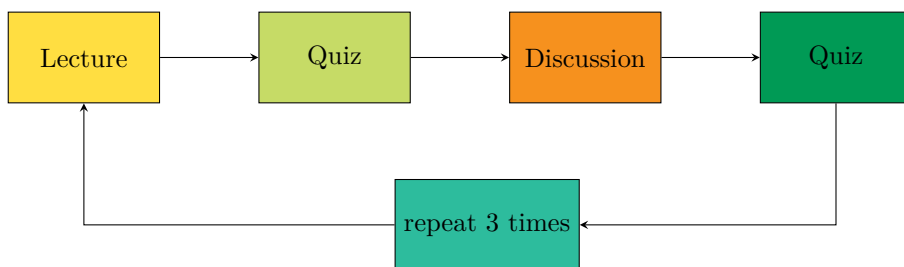


Figure 3: Variant 3

The benefit of using this variant, as in variant 1, is that the students feel compelled to pay attention as their knowledge is verified on a regular basis. Another advantage is working in groups and the ability to discuss their ways of thinking. Students practice how to convince each other and how to argue. They learn to speak and to listen to arguments simultaneously. The opportunity for an observation of active students is a decisive advantage for the teacher. The teacher can listen to the students reasoning. It is important to learn how the students explain the discussed problem in terms of their level of knowledge. It is worth to know what arguments are clear and important for them. This knowledge allows the teacher to get to know his students better and prepare more efficient and engaging classes. The effects of this method were very satisfying. The downside to this method is a certain sense of injustice. Some students may feel excluded from the discussions or not being strong enough to pursue their arguments. On the other hand, they acquire precious social skills and improve on their ways to argue. Shy students might be outperformed by the others even if they are right. These risks has to be taken into consideration by teachers applying this method at school.

4.4. Variant 4 (all in one)

The fourth variant is again based on the idea of the flipped classroom (students read some material before the class). The teacher hands out lesson teaching material some time before the class is scheduled. Students self-study the material at home. Immediately after starting the lesson, students take a four-questions quiz to assess the effects of their home learning (approximately 15 minutes were planned). The next steps follow the protocol of the third method, see Figure 4.

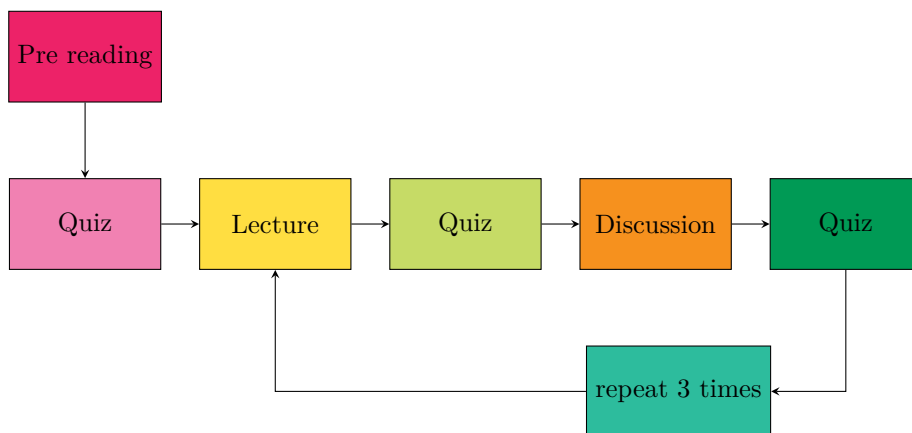


Figure 4: Variant 4

This variant is the most time-consuming one. From teacher's perspective this might be perceived as a drop-back. However, we will see in the next section that it seems to be the most effective one. In particular for each question, the number of votes for the answer with the most factual errors decreases after the discussions. The other advantages and disadvantages are mostly the same as in Variant 3. However, there are still some points worth mentioning. A very important advantage of the above observation is the possibility of checking the knowledge three times in the same group of students while working on the same material. Working initially alone at home and confronting results with peers is beneficiary for students as this process includes their emotional engagement, which in turn leads to better retention of acquired knowledge.

5. Curriculum constrains and suggestions

An important issue is to compare the timing of the variants. Table 2 shows how much time has been planned for a given topic, and how much time was actually used when working with a given variant.

Relatively big discrepancies between the planned and actually used time units can be partly explained by two factors:

Variant	Suggested by the syllabus	Actually time spent
1	45 minutes	180 minutes
2	180 minutes	210 minutes
3	45 minutes	180 minutes
4	90 minutes	200 minutes

Table 2: Comparing the planned and used time

- our inexperience in applying peer instruction in the class;
- students disorientation when encountering new method and its variations over a short period of 6 weeks.

We expect that less time is necessary when both the teacher and the students are familiar with the peer instruction approach. The relatively big discrepancies between the suggested and actually spent time were possible thanks to additional teaching hours approved by the school principal. The introduction of teaching method was motivated, similarly as in Prof. Mazur case, by the observation that students get quickly bored by the standard approach of lecture style. They were passive and not engaged during the classes. Even if our experiment was not performed in an optimal way, we observed substantial increase in students activity during the classes and an increase in their interest for the class. It is natural to wonder if this increase is temporary, sparked by the novelty of the method and would decrease after applying the method over an extended period of time. To answer this question requires a new study on which results we will report elsewhere.

In our subjective opinion, there is a need to expose students to variable ways of teaching. We showed that even a single method allows for variations and can be combined with other approaches. In our case it was the peer instruction melted with the flipped classroom. At a long term, the creativity of the teacher should result in creativity of the students. In the ideal, maybe a bit unrealistic world, they should come up with the best lessons scenarios. We are not there yet, however, in Section 6 we present our students' opinions on the experiment.

Before we pass to students' opinions, we conclude this section presenting in Table 3 a possible division of the trigonometry curriculum and we suggest variants of our method which in our opinion, based on the experience collected in classes described in this article and in some other applications of the method not accounted for here, best fit each piece of the material. The choice of the unit on triangles and trigonometric functions might seem far away from polynomials we were handling in our classes. However, there are certain similarities in formal manipulations of these objects which made us to choose this material.

We hope that these suggestions might be helpful for fellow teachers to try out our approach during their classes. We had no opportunity to try these suggestions ourselves but we hope to come back to this soon.

Variant	Sample topic
2	Rectangular triangles
2	Acute angle trigonometric functions
4	Trigonometry - applications
2	Solving right triangles
3	Relationships between trigonometric functions
3	Convex angle trigonometric functions
2	Area of a triangle
2	Area of a quadrangle
4	Summary on the whole unit

Table 3: Suggestions on the trigonometry curriculum division

6. Through the eyes of students

After completing the experiment which we called the Concept Test, the students completed a questionnaire providing their feedback about various aspects of our experiment. There were four questions. In each of them the students could choose only one answer. We present and discuss the results of the survey below.

In the **first question** we asked if an early access to the developed material helped them to better understand new topics. Most respondents - 36.7% replied that access to materials before classes facilitated their preparation for classes. Slightly less - 33.3% noticed that following the notes during the classes made it easier to understand the topic, see Figure 5.

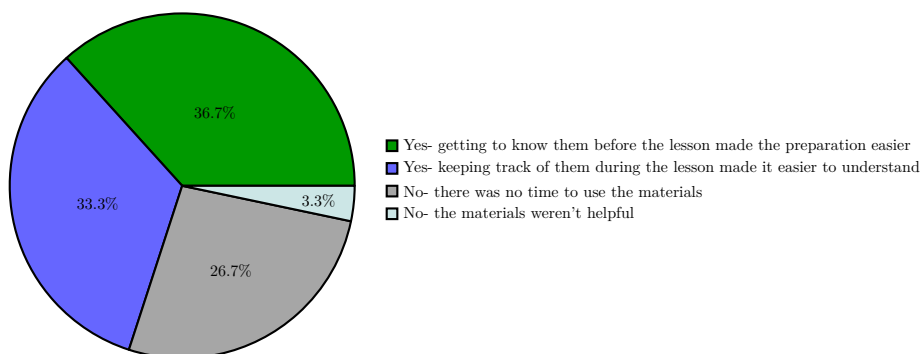


Figure 5: Students answers - question 1

From the teacher's point of view, preparing notes before class makes it easier to work with students. During the class, the teacher can focus on discussing the issues and explaining complicated information. The advantage of this idea is that

the teacher can assign more class time for important issues and is not "losing" it to introduce basic concepts.

The **second question** was the most intriguing for us. We asked if the collaborative learning in groups improved understanding of new issues. The answer to the question divided the respondents. 33.3% of them decided that they prefer to work alone whereas the group of the same size answered that it is much better to work in a group. The two other possibilities were selected by the other one third of students. 20% said that working in a group helps them to focus on the subject. The remaining 13.3% found group working distracting, see Figure 6.

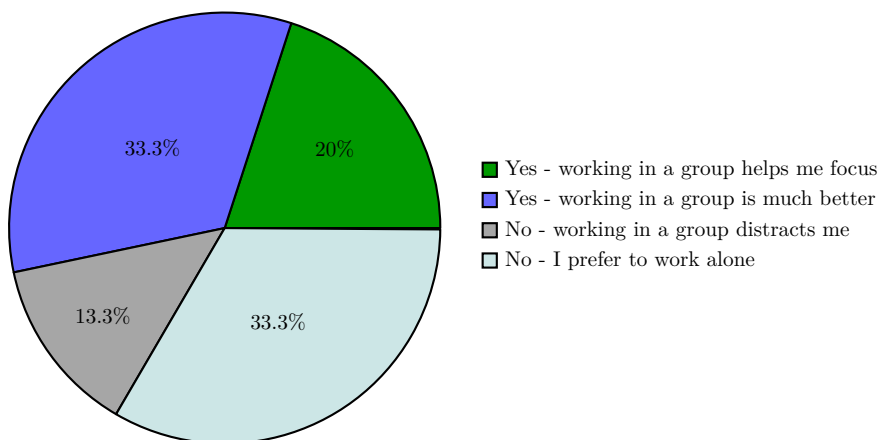


Figure 6: Students answers - question 2

All in all, a significant group of students was satisfied with the team working. It is important that they acquire this skill because this is the way that most likely most of them will work during their study and in the professional working place.

In the **third question** we asked the students about their opinion on the discussion component of the experiment and if they changed their answers based on these discussions. A relatively large group of students 73,3% answered "Sometimes", which means that at least once the discussion prompted them to change their answer. Regrettably as many as 63,3% of respondents could not remember if their change was from a wrong answer to the good answer. The groups of students who never changed their answer was of the same size as the group of students who always changed their answer. There were 13,3% of votes for each of these options, see Figure 7.

The results shows that the Concept Test questions followed by the discussion enable students to better analyse problems and see their mistakes. They find it also reassuring that many of their fellow students are not sure about the right answers neither. This provides additional motivation for looking for right answers and continuing the discussion even after the class is over.

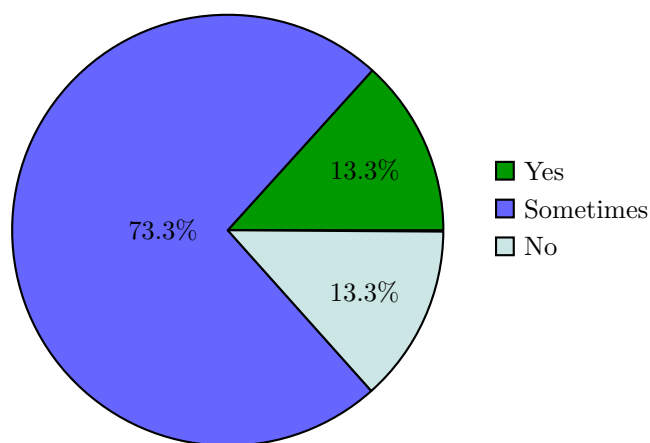


Figure 7: Students answers - question 3

In the **fourth question** we asked the students if they found it helpful to work with the Concept Tests while learning new material at school. A strong majority, 73,3% of respondents, answered this question affirmatively, see Figure 8.

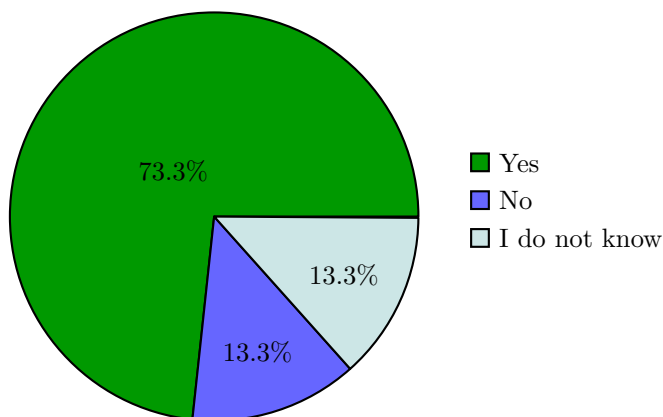


Figure 8: Students answers - question 4

Comparing working with the Concept Test and without this method lets us to formulate the following opinion. The flow of the lesson is more effective in the classes exposed to the method. The class moves from the passive position to the active one, which is visible in students participation in discussions and improved analysis of occurring problems. The students become co-creators of the learning process.

6.1. Additional feedback from the students

During the survey, the students had the opportunity to write down their own feelings and express their opinions. The following opinions occurred on the positive side of the experiment: „helping each other”, „more fun during learning”, „better communication between students”.

From the teacher’s point of view, the most satisfactory were the following answers: „if we are not sure about something, others may help us and possibly lead us out of error” and „before the class one can get a peer help, e.g. when a fellow student is better at a certain subject, it is easy to write or call him and ask.”

The students noticed also some weaker sides of the experiment: „sometimes people do not want to help, for personal reasons, e.g. when they do not like someone, or because they give the wrong answer on purpose”, „it is difficult to focus”, „not always, despite the fact that I’m right, peers listened to my arguments”.

Results and conclusions

Our experiment verifies the applicability of Mazur’s method in high school mathematics education. Passing parts of the responsibility to students and engaging them in discussions assisted by the tutor makes students more creative. We observed much better cooperation in student groups. Students understand and appreciate that working together in a group helps all of them to understand better and faster more difficult issues.

On the other hand, the instructor, following the work in groups, gains more experience himself and learns how to analyse and detect most important issues related to the part of the classroom material which is discussed at the moment. It is essential to listen to and to analyse learners points of view to improve own teaching skills and extend mastered methodology. For the role of dialog in learning see e.g. (Smit, Hess, Taras, etc., 2023). Learners who had access to materials that will be discussed during classes / lessons before they actually take place, could focus on issues that are difficult and ask questions about the more problematic passages. From the lecturer’s point of view there was no "wasting" of time for introducing, e.g., new definitions that the audience could easily read and comprehend on their own. We noticed that the effect of introducing this teaching method leads to better scoring of students at various assessments.

During the classes, we had a subjective impression of increased engagement of students, which agrees with more theoretical findings of (Crouch, Fagen, Mazur, etc., 2007). This was best visible through their increased activity, participation in discussions and visible interest in results of tests. An important factor for this was the anonymity during quick quizzes. The innovative nature of the experiment carried out lies in shifting the focus from the correct answers to the most wrong ones. The conducted research should be treated as the first step on the way to find a method that most effectively reduces the number of errors indicated by the audience in terms of content – in 21 out of 22 questions a clear decrease in the number of votes for the answer was noticed. However, this experiment should not be viewed as an attempt to find a universal teaching method working with

all classes and on all subjects. It is rather a search to enrich one's professional workshop in teaching the students more effectively. The achieved results inspire us to carry out further research.

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