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PISA, TIMSS and Swedish students' knowledge of linear equations: A 'telling' case of a system fixing something that may not be broken*

Abstract. In this paper, we construct a 'telling' case to highlight a problematic inconsistency between the results of international large-scale assessments (ILSAs) and other studies of Swedish students' knowledge of linear equations. In this context, a 'telling' case, based on the scrutiny of appropriately chosen cases, is presented as a social science counter-example to the prevailing view that ILSAs' assessments are not only valid but should underpin systemic reform. Our 'telling' case comparison of the different forms of study shows that Swedish students, in contrast with the summative assertions of the different ILSAs, have a secure and relational understanding of linear equations that persists into adulthood. We conclude with a cautionary message for the curriculum authorities.

1. Introduction

Over the last quarter of a century, international large-scale assessments, hereafter ILSAs, have become increasingly influential in national educational policy debates (Hopfenbeck, Lenkeit, El Masri, Cantrell, Ryan, Baird, 2018; Lindblad, Pettersson, Popkewitz, 2015). Such tests, which have been broadly accepted as 'valid' by policy makers" (Auld, Morris, 2014, p. 130) who have been seduced by the pseudo-science of rankings (Dossey, Wu, 2013; Pons, 2011), have prompted many governments, perceiving their students' performances as unacceptably poor, to assume that their education system must be broken and in need of fixing. One such government is Sweden, the focus of this paper, which, following repeatedly poorer than expected ILSA results, undertook major curricular revision (Sundberg, Wahlström, 2012), initiated large scale teacher professional development

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programmes (Hardy et al., 2019) and invited the OECD, the sponsor of one of the major ILSAs, to evaluate the system's structures (OECD, 2015). In so doing, it is barely surprising that the recommendations emerging from this far from impartial evaluation reflected the goals of the OECD in ways that ensured Sweden would 'see like PISA' (Gorur, 2016). In this paper, through the construction of a telling case (Marshall, 1983, 1984), we argue that Sweden's education system is not the poorly performing system highlighted by various ILSAs and, in so doing, challenge the legitimacy of the different ILSAs' results.

2. TIMSS and PISA

From the perspective of mathematics, two distinct forms of ILSA are of interest to this paper. The first, following the first and second international mathematics studies (FIMS and SIMS) (Robitaille, 1990; Brown, 1996), is the Trends in International Mathematics and Science Studies (TIMSS), which, sponsored by the International Association for the Evaluation of Educational Achievement (IEA), has been undertaken every four years since 1995. The second, the Programme of International Student Assessment (PISA), is managed by the Organisation for Economic Cooperation and Development (OECD) and has been undertaken every three years since 2000. The two forms of ILSA, which differ in their conceptualisations of subject knowledge, also have widely differing aims (Grønmo, Olsen, 2007).

TIMSS, framed against an internationally agreed but essentially hypothetical mathematics curriculum, addresses the mathematical behaviours expected of students in relation to that subject matter (Mullis, Martin, Ruddock, O'Sullivan, Preuschoff, 2009). Assessment is organised around two dimensions: a content dimension focused on the subject matter to be assessed and a cognitive dimension focused on the sorts of mathematical behaviours expected of students (Mullis et al., 2009). Its goal is to provide policy makers and educationalists with data to improve the teaching and learning of mathematics. Its organisers claim that participation in successive assessments should enable countries to identify trends in students' achievement and evaluate reforms (Foy, 2017). Thus, typically focused on grades four and eight, TIMSS aims to evaluate students' technical competence on, essentially, familiar curriculum material (Mullis et al., 2009). However, while it construes its work as supporting policy decisions, influencing policy is not a TIMSS objective.

The mathematics component of PISA, undertaken at age 15, claims to look beyond classroom mathematics towards the situations people face in their daily lives and which necessitate the application of mathematical skills in less structured contexts. It has addressed "the capacity of students to put mathematical knowledge into functional use in a multitude of different situations in varied, reflective and insight-based ways" (Schleicher, 2007, p. 351). So effective has been the OECD's promotion of PISA (Alexander, 2010; Cantley, 2019; Sellar, Lingard, 2013) that it has asserted repeatedly that it "has become the world's premier yardstick for evaluating the quality, equity and efficiency of school systems" (OECD, 2013, p. 3, OECD, 2016, p. 3), allowing governments and educators to identify effective policies that they can then adapt to their local contexts (OECD, 2013). In so doing,

and unlike the IEA's ambitions with TIMSS, it has the explicit goal of promoting policies that will improve the economic and social well-being of people around the world. Such goals, as we show below, while superficially appearing benign can take on a sinister character.

3. ILSAs, the 'grey literature' and Sweden

Since the inception of the internet, the traditional world of academic publishing has been threatened by what has become known as the 'grey literature' (Banks, 2006). Defined as "the diverse and heterogeneous body of material available outside, and not subject to, traditional academic peer-review processes" (Adams, Smart, Sigismund Huff, 2017, p. 433), 'grey literature' may incorporate "newspapers, online sites and blog posts where researchers and others try to translate their work for a general audience" (Lawrence, 2017, p. 389). In such a world, particularly when the authoritative peer-reviewed material may be hidden behind a paywall, it is not uncommon to find those in search of information turning to this 'grey' material.

The OECD has made extensive use of the 'grey literature' to publicise its results. For example, when discussing strategies for raising German politicians' awareness of PISA's results, the head of the OECD's Directorate of Education and Skills commented that

> going to the people in charge isn't going to change the system. And I actually changed strategy and thought I'm going to go to work with the media, go to work with other people, and that has created a public demand for better education... parents knocked on the door of schools, schools knocked on the door of local administrators – and a week after this the Chancellor in Germany went public about this, saying what they needed to do... (Schleicher, 2015 as cited by Grey, Morris, 2018, p. 111)

Indeed, the OECD has invested significant resources in developing its media activities to ensure that reports discussed in national media cannot go unnoticed by policy makers (Addey et al., 2017). Moreover, key to understanding the significance of this PISA-related 'grey literature' is the manner in which it has created discourses of both 'scandalisation' and, less typically, 'glorification' to warrant systemic change (Baird et al., 2016). With respect to the former, typically through the manipulation of the media to construct an image of educational crisis, scandalisation has been extensively evidenced in, for example, Germany (Ertl, 2006; Waldow, 2009), the UK (Grey, Morris, 2018) and Norway (Elstad, 2012; Nortvedt, 2018). In the latter context, the Norwegian

> media presented tabloid-like and oversimplified rankings. It seemed that the public as well as politicians accepted these versions as objective scientific truths about our education system... (thus creating) a public image of the quality of the Norwegian school that is not justified. (Sjøberg, 2015, p. 114)

That is, the 'grey literature' initiated a culture of political blame management (Elstad, 2012), which ultimately led tochanges to both the curriculum and the manner of its assessment (Hatch, 2013; Nortvedt, 2018).

By way of contrast, the discourse of glorification is exemplified in the mass of literature concerning the PISA-related achievements of Finland. For example, the media presentation of Finland's PISA-related successes has been used to legitimate various political agendas in Australia, Germany, South Korea (Takayama, Waldow, Sung, 2013) and even Japan (Takayama, 2010). Moreover, following the results of results of PISA 2000, "approximately 15,000 people from German-speaking countries visited the Finnish National Board of Education or were present when Finnish lecturers visited those countries" (Isotalo, 2004, cited in Laukkanen, 2008, p. 305). However, many of the claims concerning Finnish students' PISA-related achievements may have been over-inflated, not least because their modest performance on TIMSS was compromised by, essentially, barely average levels of competence with respect to both algebra and geometry (Mullis et al., 2000; Mullis et al., 2012). Also, a result that has gone largely unnoticed, has been the fact that Flanders, the Dutch-speaking region of Belgium, has typically out-performed Finland on both PISA and TIMSS (Andrews, 2015). Finally, there is evidence that Finnish students' PISA-related mathematics successes may be less a consequence of teaching quality than cultural characteristics unique to Finland (Andrews, 2013, 2014; Andrews, Ryve, Hemmi, Sayers, 2014).

In the context of Sweden, whose results on all iterations of both PISA and TIMSS can be seen in Table 1, a similar discourse of scandalisation to that of Norway eventually emerged, although not until after the results of TIMSS 2011 and PISA 2012. Indeed, the systemic response to the first four iterations of the two ILSAs, which reflected a natural rather than a 'scandalised' concern, was to revise the steering documents for school mathematics as part of the Swedish curriculum reform of 2011 (Skolverket, 2011a) and, in particular, address perceived inadequacies identified by the ILSAs concerning the teaching and learning of algebra (Palm Kaplan, Prytz, 2020; Skolverket, 2011b). However, the fifth iteration of the two studies showed a further decline, approximately equivalent to the PISA 2012 and TIMSS 2011 cohorts of students having received around a year's less schooling than the 2000 and 1995 cohorts[1] respectively. At this point in the national narrative, and acknowledging that the current results represented a greater decline than in any other OECD country over the same period (OECD, 2015), a 'scandalised' response emerged. For example, following the publication of the PISA 2012 results, the newspapers Dagens Nyheter and Aftonbladet asserted, respectively, that "Sweden is the worst in the class" (Sverige sämstiklassen)[2] and "the Swedish school is

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[1]According to Jerrim and Shure (2016) the OECD operates under a rule-of-thumb that 30 PISA points approximates one year of schooling.

[2]https://www.dn.se/nyheter/sverige/sverige-samst-i-klassen/

sinking" (Svenskskolas junker)[3]. Even the Royal Swedish Academy of Engineering Sciences engaged in the process, publishing a report in 2016, titled, Educational performance in Swedish schools is plummeting – what are the facts? (Henrekson & Järvall, 2016). Moreover, in addition to both the government and the opposition independently developing policies to counter PISA-induced perceptions of educational decline (Ringarp, Rothley, 2010), the authorities invited the OECD to investigate the state of Swedish education. Its report concluded that "Sweden should implement a comprehensive education reform to bring about system-wide change and strengthen the performance of all Swedish schools and students" (OECD, 2015, p. 8). Moreover, as a result of the OECD's intervention, the authorities initiated a School Commission charged with developing plans for the improvement of teaching, learning and equity (Grek, 2017; Hardy et al., 2019; Löfstedt, 2019; Wahlström, Sundberg, 2018). Interestingly, the Commission's interim report (SOU, 2016:38, p. 13) began with the assertion, tacitly acknowledging the validity of the ILSA results, that "according to international surveys, the learning outcomes of Swedish schools have been in decline for several decades" (our translation). Moreover, to an extent not found elsewhere, both public and politicians, motivated by the 'scandalised' media and Schleicher's comments about the Swedish school system having lost its soul[4], coalesced around a widespread acceptance of the OECD's importance and indispensability as an education policy advisor (Grek, 2017). Indeed, this coalescence was further evidenced by the fact that the OECD review was invited by a right-of-centre government, while the School Commission was initiated by a left-of-centre government (Wahlström, Sundberg, 2018). In sum, by inviting the OECD investigation and instigating the school commission, politically different Swedish authorities not only accepted the validity of PISA's assessments but conceded that the future of Swedish education lay in 'seeing like PISA' (Gorur, 2016).

TIMSS grade 8	1995	1999	2003	2007	2011	2015	
mathematics	519	N/A	499	491	484	501	
PISA	2000	2003	2006	2009	2012	2015	2018
Mathematics	510	509	502	494	478	494	502

Table 1: Swedish students' mean scores on all PISA and TIMSS undertaken[5]

That being said, in framing this paper, we feel compelled to ask whether the scale of Sweden's PISA and TIMSS achievement declines was feasible? Was it possible that Swedish education had become so poor that, over the course of little more than a decade, students who once performed comfortably above the international mean were now performing equally far below? Were there underlying reasons that would have negated the need for, say, the OECD review, ILSA-prompted curricular changes or the School Commission? In this paper, and acknowledging that

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[3]https://www.aftonbladet.se/nyheter/a/8wOvEr/svenskskola-sjunker
[4]https://www.theguardian.com/world/2015/may/04/sweden-school-choice-education-decline-oecd
[5]In general, both TIMSS and PISA report their results against an international mean of 500 and a standard deviation of 100.

the Swedish authorities had identified algebra as an issue of concern, we examine Swedish students' knowledge of linear equations as evidenced by PISA, TIMSS and various independent studies. To do this, as we outline below, we aim to construct a 'telling' case (Mitchell, 1983, 1984) that will act in the manner of a social science counter example to challenge not only the validity and relevance of both ILSAs' assessment of Swedish students' understanding of this key topic but also the Swedish authorities' invitation to the OECD.

4. The 'telling' case

Mitchell, who died in 1995, was a social anthropologist who construed his field as having "been built up (...) from a large number of separate case studies" enabling anthropologists "to draw inferences and to formulate propositions about the nature of social and cultural phenomena in general" (Mitchell, 1983, p. 189). In this paper, and following Mitchell's lead, we marshal a series of independent cases to construct a 'telling' case (Mitchell, 1984) to serve as a social science equivalent of a counter-example to the prevailing view that ILSAs are relevant to Swedish education. Broadly speaking, through the identification of cases likely to "illuminate formerly obscure aspects of the general theory", the 'telling case' isolates "the necessary circumstances for the manifestation of some phenomenon" (Mitchell, 1983, p. 202). In Mitchell's view, "the search for a 'typical' case (...) is likely to be less fruitful than the search for a 'telling' case in which the particular circumstances surrounding a case serve to make previously obscure theoretical relationships suddenly apparent" (Mitchell, 1984, p. 239). Indeed, in essentially dismissing the typical case, he discusses the 'significance of the atypical case' (Mitchell, 1983, p. 203) and the manner in which it offers 'instances where the concatenation of events is so idiosyncratic as to throw into sharp relief the principles underlying them', providing, of course, the researcher is sufficiently familiar 'with current theoretical formulations' as to be able to recognise such concatenations for what they are (Mitchell, 1983, p. 204).

So, how does Mitchell's notion of a 'telling case' underpin a social science counter-example and what, in the context of social science, could be construed as a counter-example? According to Mitchell, a 'telling case' is based on the scrutiny of appropriately chosen cases (Mitchell, 1983). A 'telling case', by implication, draws on more than a single example that could otherwise be dismissed as coincidence. It requires the unexpected coalescing of different forms of evidence whose connections had hitherto gone unnoticed. In this way, the identification of different sources, each of which offers a similar perspective on the issue under scrutiny, can be construed as a 'telling case' and, if that 'telling case' challenges the received view of that issue, then it can be considered a counter-example, albeit one without the rigorous consequences of its mathematical equivalent. In constructing our 'telling case', we acknowledge not only Mitchell's (1983, p. 202) appeal for analytical induction and the need "to specify the necessary connections among a set of theoretically significant elements manifested in some body of empirical data", but also that when

setting out a case study, the analyst must decide in advance at what point to enter the ongoing flow of events and at what point to withdraw from it. For the purposes of exposition, a set of events must be lifted from the ongoing stream and presented, as it were, isolated from antecedent and subsequent events. (Mitchell, 1984, p. 237)

In this paper, in order to lift events from the ongoing stream, we draw on various independent sources to highlight inconsistencies in both PISA's and TIMSS' assessment of Swedish students' knowledge of linear equations. Overall, the 'telling case' that emerges from these different events is not only deeply contextual, a key element of the 'telling case', but also makes "previously obscure theoretical relationships suddenly apparent" (Mitchell, 1984, p. 239). That being said, while a social science counter-example of this form will never have the same authority as its mathematical equivalent, the critical mass of independently produced data on which the 'telling case' is based validates a challenge to a received educational truth.

5. Linear equations in the Swedish national curriculum

The Swedish national curriculum, which is structured by the school years 1–3, 4–6 and 7–9, asserts that by the end of year 3 students will understand "mathematical similarities and the importance of the equals sign" (Skolverket, 2011a, p. 60), which is an understanding necessary for the solving of algebraic equations. It adds, more explicitly in relation to linear equations, that by the end of year 6, students will be familiar with "unknown numbers and their properties and also situations where there is a need to represent an unknown number by a symbol; simple algebraic expressions and equations in situations that are relevant for pupils; methods of solving simple equations" (Skolverket, 2011a, p. 61). Finally, by the end of year 9 students will understand the meaning of the concept of variable and its use in algebraic expressions, formulae and equations; algebraic expressions, formulae and equations in situations relevant to pupils; methods for solving equations; functions and linear equations" (Skolverket, 2011a, p. 63). Importantly, the significance of which will become clear below, there is no systemic expectation in the compulsory school curriculum for students to engage with quadratic equations, although there remains the possibility that some may have experienced them prior to their ILSA assessments.

With respect to post compulsory education (typically known as upper secondary school, hereafter, USS), Swedish students opt for one of eighteen tracks, each of three years' duration. These comprise 12 vocational and six academic tracks. Depending on their track choice, students may study up to six mathematics courses, each of one semester's duration and representing an increasing sophistication. That being said, all students, irrespective of track, are obliged to follow at least the first of these courses, which is designed to complement and extend students' earlier mathematical experiences (Larson, Bergsten, 2013) and includes further exposure to linear equations. Thus, by the time they complete USS, all Swedish students would have had several exposures to linear equations. At compulsory school, due to the loose framing of the Swedish curriculum, some

students would have met linear equations before TIMSS in grade 8, others between TIMSS in grade 8 and PISA at age 15, others after PISA, with still others experiencing more than one such exposure. At USS, the compulsory first course confirms that all students would have met the topic at least one more time.

6. Identifying the 'telling' case: PISA

While every iteration of PISA incorporates an assessment of mathematics, mathematics was the principle focus of PISA 2003 and PISA 2012. In the following, principally due to Sweden's 'scandalised' response, we focus solely on PISA 2012. Importantly, when the results are scrutinised, particularly with respect to algebra in general and linear equations in particular, inconsistencies emerge that raise questions about the integrity of PISA's assessment of Swedish students. It is also important to remind the reader that PISA assesses students at age 15, which typically means the assessed Swedish students would be in grade nine, the final year of compulsory school and the same year of their national test.

Due to mathematics being its principal focus, PISA 2012 included six self-report measures of student's opportunity to learn, hereafter abbreviated to OTL. Of these, three had elements specifically involving equations in general and linear equations in particular that, collectively, create a confusing picture of Swedish students' knowledge of the topic. It is on these that the following focuses. The first OTL question comprised nine items, each set against the words, "how often have you encountered the following types of mathematics tasks during your time at school?" and the headings, frequently, sometimes, rarely or never (OECD, 2013, p. 170). Of these nine questions, three addressed equations, two of which were quadratic and one linear. These were $6x^2 + 5 = 29$, $2(x + 3) = (x + 3)(x - 3)$ and $3x + 5 = 17$. While the OECD's (2013) report includes no results for the third of these items, which is unfortunate given the context of this paper, the results for the first two items are shown in Table 2. It can be seen that around four-fifths of Swedish students claim, somewhat bizarrely, a not infrequent exposure to a topic they are not expected to have been taught.

Solving an equation like:	Freque-ntly (%)	Some-times (%)	Rarely (%)	Never (%)	OECD (2013) page reference
$6x^2 + 5 = 29$	45	38	11	7	353
$2(x+3) = (x+3)(x-3)$	42	38	13	7	355

Table 2: Swedish students' self-reported exposure to two forms of quadratic equation

The second OTL question compounds the confusion. It was based on thirteen items set against the phrase, "thinking about mathematical concepts: how familiar are you with the following terms?" and five categories of response (OECD, 2013, p. 170). Three of these, which formed the basis of a summative measure of their familiarity with algebra, concerned students' familiarity with exponential functions, quadratic functions, and linear equations (OECD, 2013, p. 173). As indicated above, of these three topics, only linear equations is specified in the Swedish curriculum for compulsory school. Thus, any measure based on such items is likely to skew their performance towards the lower end of the international scale. Indeed, the results for these three items, summarised in Table 3, show Swedish students claiming very limited familiarity with all three topics. This apparent lack of familiarity is of interest for at least two reasons. The first is that their familiarity with linear equations seems exceptionally low. For example, referring to the international mean, the OECD report commented that "only 42% of students in OECD countries reported that they know linear equations well" (OECD, 2013, p. 161), indicating, in the use of the word *only*, that this was an unexpectedly low proportion. However, if the OECD mean was unexpectedly low then the corresponding figure for Sweden, 8.6 seems barely credible and, if taken seriously, could be construed as a national embarrassment. The second concerns students' claims of regular exposure to quadratic equations (at least in the context of the two items in Table 2) but almost no familiarity with quadratic functions (at least in the context of Table 3). Admittedly, a quadratic function is mathematically different from

	Never heard of it (%)	Heard of it once or twice (%)	Heard of it a few times (%)	Heard of it often (%)	Know it well, understand the concept (%)	OECD (2013) page reference
Exponential function	71.2	16.1	6.6	2.9	3.1	361
Quadratic function	59.7	19.6	11.6	4.5	4.6	363
Linear equation	39.0	25.5	17.5	9.4	8.6	364

Table 3: Swedish students' self-reported familiarity with three topics of school algebra

a quadratic equation, so caution should be exercised in any interpretation of these figures, but the seemingly inexplicable discrepancies may be explicable. It is known that Swedish students at the time of PISA will have experienced linear but not necessarily, as indicated earlier, quadratic equations. Therefore, it is conceivable that they may have interpreted $6x^2 + 5 = 29$ and $2(x+3) = (x+3)(x-3)$ as examples of material with which they are familiar and, therefore, claimed to recognise them. Also, such false recognition of the symbolic form may be due to the vocabulary used by PISA in comparison with that of the Swedish national curriculum; the Swedish versions of PISA's tests include references to second degree equations and

second degree functions (andragradsekvationer and andragradsfunktioner), while the documents of the national curriculum, which include only expectations of linear functions and equations, include only the nouns equation and function with no qualifying adjective.

The fourth OTL question addressed two mathematical skills, each presented in two contexts. The contexts were framed against the questions (OECD's emphases), "How often have you encountered these types of problems in your **mathematics lessons**?" and "How often have you encountered these types of problems in the **tests you have taken at school?**". The two skills focused on solving the equation $2x + 3 = 7$ and calculating the volume of a box with sides 3m, 4m and 5m. However, as far as can be discerned from its report, the OECD combined the results of the four possible responses into one score (OECD, 2013, p. 357). That being said, the figures for Swedish students show, with respect to the headings of frequently, sometimes, rarely and never, percentages of 62.9, 32.3, 3.4 and 1.4 respectively. That is, assuming the score is some average of two separate scores, it would seem that 96% of Swedish students are not infrequent experiencers of linear equations, albeit equations with the unknown on one side of the equals sign only. The scale of these results supports the conjecture above that Swedish students are familiar with symbolic representations of equations but not the word linear.

Finally, with respect to PISA's assessment of formal mathematics, the results seem to add to the confusion. The formal mathematics scale is an amalgam of three scales. The first of these is the familiarity with algebra scale, based on the three items derived from question two. The second is a geometry scale based on a different set of four items from question two, while the third was based on the results of question four above. Swedish students' score on this dimension, 0.77 (OECD, 2013, p. 347), was by some distance, the lowest in the world, being the only score less than one and considerably below half that of the OECD mean of 1.70. In this respect, the extent to which Sweden appears an outlier on this dimension is well demonstrated in the graph found on page 169 of the OECD's report. However, such a position for one of the world's leading economies seems implausible. Whatever the reason for this seemingly bizarre state of affairs, it seems difficult to accept the measures presented above as valid assessments of Swedish students' familiarity with and knowledge of algebra in general and linear equations in particular.

7. Identifying the 'telling' case: TIMSS

Unlike PISA, all TIMSS assessments have focused equally on mathematics and typically addressed the four broad topic areas of number, algebra, geometry and, as one domain, data and chance. Over the five iterations of TIMSS on which Sweden has participated, shown in Table 4, Swedish grade eight students' overall mathematics performance has been variable. However, in every case, the overall figure is a consequence of (relatively) high performance on number and data, and (relatively) low performance on algebra and geometry.

While the topic of linear equations forms a core element of the TIMSS assessment framework, being explicitly acknowledged at intermediate, high and advanced achievement levels (Mullis, et al., 2016), its visibility in the official reports has been

Year	Overall	Number	Algebra	Geometry	Data
2015	501	513	482	478	512
2011	484	504	459	456	504
2007	491	505	459	472	526
2003[6]	499	496	480	467	539
For TIMSS 2003 onwards, the figure for each section was based on an expected mean of 500 and standard deviation of 100					
1995	519	62	44	48	70
For TIMSS 1995, the figure for each section was reported as the percentage of correctly answered items					

Table 4: Swedish students' TIMSS results

limited. In this respect, Table 5 shows the relevant and, importantly, available linear equations-related tasks gleaned from both official reports and sets of released items when accompanied with the appropriate statistics. While the formatting of the tasks has been adapted for this paper, the phrasing remains exactly as in the original. In general, we have not included tasks focused on simultaneous linear equation, usually presented in multiple-choice formats, but tasks interpretable as addressing the solution of linear equations.

As a benchmark for what transpired later, the single 'naked' equation in Table 5, presented in TIMSS 1995 as a multiple-choice task, yielded a 51% competence level for Swedish students against an international mean of 72% (Beaton, Mullis, Martin, Gonzalez, Kelly, Smith, 1996, pp. 74, 77). Interestingly, the released items for TIMSS 1995 also included the task, find x if $10x - 15 = 5x + 20$ from which additional insights could have been inferred. Unfortunately, no statistics can be found for this item. With respect to TIMSS 2003, no linear equations-related tasks are inferable from the available data, although several such tasks had been released[7], albeit without relevant statistics.

TIMSS 2007 offered two tasks, shown in Table 5, of relevance to this paper. The first task, a multiple choice, involved the solution of an equation presented as a word problem. Students were asked to determine how many items could be shipped for a sum of 150 zeds (a zed is the de facto currency unit for all TIMSS studies) if the cost of shipping, y, was equal to $4x + 30$, where x is the weight in grams per item. Here, being expected to substitute a value for y before solving $150 = 4x + 30$, Swedish students achieved a mean of 23% compared with an international mean of 34%. The second task, presented in the report as "a word problem that can be expressed as two linear equations with two variables" (Mullis et al., 2008, p. 102), was written in such a manner that it would just as easily yield a single equation in one unknown. On this task, which reduced to solving $5x + 2 = 17$, Swedish students achieved a mean of 34% against an international mean of 18%.

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[7]https://timss.bc.edu/timss2003i/released.html

The two tasks discernible from TIMSS 2011 and TIMSS 2015 are conceptually similar, with both inviting students to form an equation from the sum of three lengths, each represented in terms of an unknown. The tasks yield the equations $4x + 8 = 40$ and $4x + 10 = 30$ respectively. Neither is conceptually challenging and both could have succumbed to an operations reversal or even trial and improvement. However, while the 2015 task specifically invites students to devise an equation before, tacitly, inviting them to solve it, the 2011 task is less transparent. Interestingly, the proficiency level of Swedish students on the second task was more than three times that of the first.
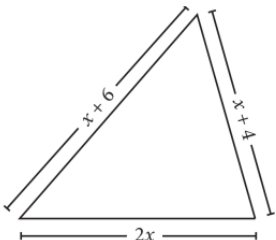
Year	Task	Sources	Sweden (% correct)	World (% correct)
1995	If $3(x + 5) = 30$ then $x =$ A. 2 B. 5 C. 10 D. 95	Beaton et al., 1996, pp. 74, 77	51	72
2007	In Zedland, total shipping charges to ship an item are given by the equation $y = 4x + 30$, where x is the weight in grams and y is the cost in zeds. If you have 150 zeds, how many grams can you ship? A) 630 B) 150 C) 120 D) 30	Mullis et al., 2008, p. 106	23	34
2007	Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?	Mullis et al., 2008, p. 102	34	18
2011	A piece of wood was 40cm long. It was cut into 3 pieces. The lengths in cm are $2x - 5$, $x + 7$, $x + 6$. What is the length of the longest piece?	Foy et al., 2013b, p. 32, Foy et al., 2013a, p. 21	8	11
2015	The sum of the lengths of the sides of this triangle is 30 cm. A. Write an equation that would enable you to find the value of x. B. What is the length of the LONGEST side of the triangle in centimetres?	Mullis et al., 2016, Exhibit 2.14.3	27	22

Table 5: Linear equations tasks extracted from the different iterations of TIMSS

Overall, what can be inferred from the different TIMSS results? Firstly, adopting the scandalisation perspective, it could be argued that Swedish students' ability to construct and solve equations is limited in relation to their international peers. However, such a conclusion ignores that fact that Swedish grade eight students' exposure to equations would be dependent on the textbooks their teachers use; some will have experienced equations and others not. Secondly, the different tasks seem to invoke different forms of response. This is particularly interesting in light of the two TIMSS 2007 tasks. Here, the 'shipping charges' task, which is construed as a simple linear equation requiring an initial substitution, is construed by the TIMSS task developers as conceptually less challenging than the 'pen and pencils' task, which was construed as a simultaneous equations problem. Yet, Swedish students performed substantially below the international mean on the former (Mullis et al., 2008, p. 106) and substantially above the international mean on the more conceptually challenging latter (Mullis et al., 2008, p. 102). What makes this result more surprising is the fact that the result on the former is lower than would have been expected if all students had adopted a random approach to the multiple-choices available to them. Further, possibly undermining the validity of the 'pen and pencils' task as an assessment of simultaneous equations, is the possibility that Swedish students, with no prior experience of such mathematics, were better able than their international peers to arrive at a correct solution.

Similar inferences can be drawn from the conceptually similar tasks found in TIMSS 2011 and 2015. On the one hand, Swedish students' achievement on the 'piece of wood' task was significantly lower than the international (Foy et al., 2013a, p. 21), while that of the 'triangle' problem was significantly higher (Mullis et al., 2016, Exhibit 2.14.3). Admittedly, the rubric of the 'triangle' task invited students to construct an equation before solving it and offered, also, the advantage of a diagram to indicate the relationship of the three given lengths. However, these differences applied to all students and may explain the higher success rates generally. They would not explain the directional change in Swedish students' achievement. In sum, while these TIMSS items may appear to offer a more transparent assessment of students' equations-related competence than the more speculative items of PISA, they yield some seemingly inexplicable results with respect to how Swedish students address them.

8. Developing the 'telling' case: three studies of grade nine students

The two ILSAs discussed above present ambivalent perspectives on Swedish students' understanding of and competence to solve linear equations. Indeed, so ambivalent are the results of both studies that it seems reasonable to infer that neither PISA nor TIMSS offers anything useful, not least because PISA's assessment is so confusing and TIMSS comes too early and may be prone to 'trial and improvement' solutions that undermine their topic-related credibility. In the following, in order to clarify the situation with respect to Swedish students' equations-related knowledge, we develop the 'telling' case through an examination of a range of independent sources. To do this we turn, initially, to three independently conducted

studies involving representatively large groups of students, assessed either shortly before the end of compulsory school or shortly after.

The first study draws on data from the Swedish national tests of achievement for students in grade nine. Periodically, details of those tests are released and the figures of Table 6 show the results for three equations-related items included in the national tests for 2010 – before PISA 2012 and TIMSS 2011 – and 2013 – after PISA 2012 and TIMSS 2011.

Year	Task	Success (%)
2010	Solve the equation $13 - 3x = 7$	67
2013	Solve the equation $\frac{1}{2}x + 1 = 5$	86
2013	Solve the equation $2(x+1) = 5 - 2x$	30

Table 6: Equations-related success rates from the Swedish grade nine national test

The second study, essentially an assessment of the knowledge students bring from their experiences of compulsory school, draws on data from the diagnostic test given to all students transferring to a Stockholm upper secondary school. Each year several thousand student sit the test at the start of the school year and before they receive any teaching. In terms of its timing, it occurs shortly after the national tests and is construed as an indicator of what knowledge students retain over the interim summer period. This, too, includes equations-related tasks, the results for which over a five-year period can be seen in table 7.

	2011	2012	2013	2014	2015
Solve $2x + 7 = 25$	86	86	85	84	86
Solve $5x - 8 = x + 3x - 8$	35	30	31	30	34

Table 7: Equations-related success rates from Stockholm city's USS diagnostic test

The outcomes of these two forms of test are remarkably complementary. First, a simple equation with the unknown on one side and a positive coefficient always elicits a success rate in the mid-80s. Second, with respect to the national test alone, a simple equation involving a single unknown with a negative coefficient reduces that average but still indicates (67%) relatively high proficiency. Thirdly, a more complex equation with the unknown on both sides sees the proficiency rates falling to the low 30s on both tests.

The third study draws on data from an independently conducted examination (Petersson, 2018) of native-born and immigrant students' solutions of the equation, $2x+3=11$. He found that 68% of his sample of 259 grade nine students were able to solve the equation correctly, a figure that rose to 72% once recently immigrated students, with limited access to Swedish, were removed from the analysis. However,

Petersson's sample drew on "schools with a high proportion of immigrant students" (p. 182), a characteristic typically associated with relatively low level of socio-economic status and lower educational attainment than schools in general. In such circumstance, his results can be construed as comparable to those of the two larger studies.

The totality of these three studies seems to challenge the received view, prop-agated by the ILSAs, whether implicitly or explicitly, that Swedish students are among the world's least familiar with the topic? Indeed, they indicate high lev-els of procedural competence, at least with regard to simple equations with the unknown on one side. However, these studies offer no more insight into students' solution strategies than the different ILSAs, with the consequence that little can be inferred about their conceptual understanding, although it seems clear that students' equations-related competence does not diminish over the summer break between the end of compulsory school and the beginning of upper secondary.

9. Consolidating the 'telling' case: two studies of older students

So far, having identified linear equations as a topic of some concern with re-spect to Swedish students' ILSA-related achievement, we have shown how students' performance on the Swedish national tests and Stockholm's diagnostic test for USS present a very different picture of mathematical competence. Unfortunately, nei-ther of these tests offers any indication of students' equations-related reasoning, although Petersson's (2018) study acknowledges the issue in a limited manner. Therefore, it seems reasonable to examine, as an integral element of the 'telling' case, studies that go beyond the answer only strategy of the studies reported above but also those that consider students' equations-related competence beyond compulsory school. In the following, we present summaries of a qualitative study undertaken towards the end of the first year of USS and a quantitative study un-dertaken with beginning primary teacher education students. Both studies offer additional insights to suggest that ILSA assessments, typically undertaken prior to the end of compulsory school, of Swedish students' equations-related competence may be sending false messages to the authorities.

The first of these studies (Andrews, Öhman, 2019) involved 12 group interviews with 39 USS students from two different Stockholm schools. Students, in the second semester of their first year of USS, were presented with the solution shown below.

$$x + 5 = 4x - 1$$
$$5 = 3x - 1$$
$$6 = 3x$$
$$2 = x.$$

Presented with no annotations to help them interpret the solution, students were asked to imagine they had a friend who had been absent when such equations were introduced at school and to consider how they would explain the solution to help their friend understand what is happening. The study found not only that all students recognised the equation and how it had been transformed but typically discussed the nature of the missing value or unknown. Further, in nine of the

twelve cases students volunteered a 'do the same to both sides' (DSBS) narrative to support their friend's learning. In the remaining three cases, a 'swap the side, swap the sign' (SSSS) perspective was superseded by DSBS due to concerns that a reliance on the former was likely to mask their friend's understanding. Also, albeit interviewer-prompted, students construed the balance scale as a powerful metaphor for underpinning a conceptual understanding of the equation solving process. Thus, acknowledging that few Swedish students do not continue to USS after compulsory school, this study suggests that during their USS years, students acquire a mathematically robust understanding of linear equations sufficient to recognise and provide a narrative to the solution of an equation with the unknown on both sides.

The second study (Andrews, 2020), drew on first year primary teacher education students' individual written responses to the same task as used in Andrews and Öhman (2019). Presented at the very beginning of their programme, students were asked to write a response to their absent friend. Unlike the USS students discussed above, many of these were drawing on experiences from several years earlier. Indeed, the mean age of the 156 students involved was 25, the median was 23 and the oldest was 48. In other words, exposure to linear equations was unlikely to have been recent for the majority of participants. The data, which were coded against an iteratively framework for cross-cultural comparisons (Andrews, Larson, 2019), yielded no significant differences with respect to either age or gender, showing consistent perspectives on the equation and their support of their fictitious friend. Andrews found most students (69%) expressing clear objectives for the equation solving process concerning identifying the value of the unknown. Moreover, while a minority (33%) wrote a narrative referring to SSSS, the majority (62%) wrote something attributable to DSBS. Further, of the 52 students who wrote something relating to SSSS, 19 warranted that use by reference to DSBS. In sum, the majority of students, including those who would have had no exposure to equations for several years, had a clear understanding of the purpose of equation solving and a procedure underpinned by a conceptually strong DSBS.

10. Discussion

In this paper, we set out to establish the basis of a 'telling' case (Mitchell, 1984) that would challenge the relevance and validity of both PISA and TIMSS for Swedish education. In constructing the 'telling' case, whereby we look to uncover previously hidden theoretical relationships, we have focused on a single mathematics topic, linear equations, as a counter-example to the assertions that both ILSAs undertake valid assessments designed to facilitate benchmarking, both across cultures and over time within cultures. The choice of linear equations was no accident. It has an important transitional function at the point when school mathematics shifts from concrete and inductive to abstract and deductive (Andrews, Sayers, 2012) as well as acting as a *gatekeeper* between school mathematics and higher education and employment (Knuth, Stephens, McNeil, Alibali, 2006).

To solve a linear equation requires a variety of related understandings and competences, not least of which is that students understand that the purpose of equation solving is to identify systematically the values of an unknown that establishes the equality of the two sides of an equation. In so doing, it requires a relational understanding of the equals sign (Alibali, Knuth, Hattikudur, McNeil„ Stephens, 2007). That is, it should be seen as representing equality between two expressions and not a command to operate (Falkner, Levi„ Carpenter, 1999; Mc-Neil et al., 2006). It requires learners to understand and manipulate the symbols in which equations are represented (Huntley, Marcus, Kahan, Miller, 2007). Thus, solving an equation requires not only that learners "understand that the expressions on both sides of the equals sign are of the same nature" (Filloy, Rojano, 1989, p. 19) but also that they are able to operate on the unknown as an entity and not a number (Kieran, 2007). While these various competences have been known for many years, they have been effectively ignored in the typical ILSA assessments, which have been concerned only with the correctness of students' answers, answers that have presented a largely incoherent perspective on Swedish students' familiarity with and understanding of linear equations.

By way of contrast, the 'telling' case evidence, collected from a variety of independent sources seems clear. At around the time they complete compulsory school, whether before or after their summer holidays, Swedish students demonstrate high levels of technical competence with respect to simple linear equations with positive coefficients and the unknown on one side of the equation only. With respect to more complex linear equations with unknowns on both sides their competence falls to around one in three students. This evidence is unequivocal. Later, part way through their experience of USS, students acquire a conceptually strong understanding of linear equations that includes a clear of objective and procedures typically based on a principled grounding of DSBS and an awareness of the relational role of the equals sign. Even among the minority who advocate a rote SSSS, a not insignificant proportion also invoke DSBS, indicating that their use of SSSS may be conceptually underpinned by DSBS. Moreover, these competences remain after students leave school, sometimes many years later. In addition, which is one reason why delving into students' conceptions of the equation solving process is important, the results of these latter two studies suggest that Swedish teachers are likely to have employed the balance scale as a didactical tool, which has been used internationally as a means of facilitating a relational understanding of the equals sign and warranting a DSBS procedure (Andrews, Sayers, 2012; Araya et al., 2010; Caglayan, Olive, 2010; Vlassis, 2002; Warren, Cooper, 2005).

Furthermore, the equations-related competence of Andrews' teacher education students brings an additional dimension to the 'telling' case. For example, in contrast with the limited findings from elsewhere, whereby Turkish and American teacher education students were found to have an underdeveloped understanding of the conceptual basis of equations (Isik, Kar, 2012; Casey, Lesseig, Monson, Krupa, 2018), the same could not be said of the majority of these students. Indeed, the equations-related conceptions of these Swedish teachers were not dissimilar to the limited evidence of Tossavainen, Attorps, Vaisainen's (2011) study, of Finnish, South African and Swedish university students, which, unfortunately, typically ag-

gregated the data from all three nations. Moreover, Andrews' students, enrolled on a primary teacher education programme, participated at the start of their programme prior to any mathematics-related interventions, whereas both Isik and Kar's and Casey et al.'s students, enrolled on secondary mathematics teacher education programmes, were studied at the end of their programmes. In other words, the sophistication of the students of Andrews' study confounds what might have been expected on the basis of evidence gleaned from elsewhere.

All the above support at least four pertinent conclusions. The first, particularly in light of the Stockholm diagnostic test, is that Swedish students' equations-related competence has remained constant over time. The second is that a test based on material students may not have covered is unlikely to prove useful as a benchmark of a system's success (Bautier, Rayou, 2007; Labaree, 2014). The third, which is not unrelated to the second, is that the mismatch between the ILSA and the 'telling' case assessments of students' equations-related knowledge suggests that little credence can be afforded the ILSA conclusions and that any 'scandalised' response to the different ILSA results is disingenuous. The fourth is that basing policy decisions on ILSA-related evidence is unlikely to be in Sweden's long-term interest. In essence, while ILSAs allude to a failing system, the 'telling' case suggests the opposite and supports an argument that Sweden would be better served by basing policy decisions on internally derived data garnered at the point at which students complete USS. Indeed, were Andrews' study of teacher education students construed as a delayed post-test, then Swedish students' understanding of equations not only resists the erosion of time but also presents an indicator of a successful system. Moreover, when Swedish grade eight students encountered simultaneous equations, a topic they would not have experienced, their ability to find a solution, which students internationally were typically unable to do, should be seen as a systemic success.

This notion of Sweden as a successful system not only frames our closing thoughts but prompts an important but provocative question for the Swedish authorities. If, as the ILSAs seem to suggest, Sweden is an educationally failing system, how is the country's repeated success on measures of economic competitiveness and innovation explained? The biennial reports of the World Economic Forum have shown, for at least as many years as the OECD has been promoting PISA, that Sweden is one of the most economically competitive countries and continues to produce world-leading innovators (Porter, Schwab, 2008; Schwab, 2010, 2012, 2014, 2016; Schwab, Porter, 2006; Schwab, Porter, Sachs, 2000, 2002). Also, further confounding the ILSA results, a recent interview study of Swedish upper secondary students' perspectives on the purpose of school mathematics, found sophisticated levels of economic awareness, the sort of knowledge privileged by PISA, at both personal and societal levels (Nosrati, Andrews, 2017). Finally, the Swedish curriculum, at all levels, expects student to acquire the personal responsibility necessary for successful democratic participation. In this regard, it is telling that in both 2003 and 2012 the OECD, as a largely unreported component of its PISA assessments, undertook an evaluation of what it called the effort thermometer. This comprised two questions; "how much effort did you put into doing this PISA test?" and "how much effort would you have invested if your marks from the test

were going to count towards your school marks?" (Skolverket, 2015, p. 18). Analyses showed that while Swedish students' effort with respect to tests that matter personally remained consistently among the highest internationally, their espoused effort with respect to PISA itself fell significantly over time, to the extent that by 2012 the difference between their two scores was the largest in the world (Skolverket, 2015). In other words, PISA's own data confirm, as with other studies, that Swedish students take ILSAs less seriously than their international peers (Eklöf, 2007; Eklöf, Pavešič, Grønmo, 2014), a phenomenon that should be regarded less as a systemic failure than a curricular triumph; they know when a test matters and act accordingly.

So, has the 'telling' case been established and, if so, has it isolated "the necessary circumstances for the manifestation of some phenomenon" (Mitchell, 1983, p. 202)? Our view is that the independent but topically connected evidence above has highlighted a major mismatch between two realities; that of the ILSAs and that of independent researchers. Over the course of their school experiences Swedish students acquire and retain for some years afterwards a sophisticated understanding of linear equations at odds with how the ILSA snapshots perceive matters. Thus, our 'telling' case shows clearly that Sweden should engage with ILSAs cautiously; their analyses may lead to decisions that counter the national interest and, worryingly, replace currently well-functioning structures with a set of new clothes for the Emperor. In other words, having been persuaded to see like PISA (Gorur, 2016), Sweden may have been duped into trying to fix something that may not be broken.

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