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To the editor, Annales Universitatis Paedagogicae Cracoviensis

In Błaszczyk, 2018 Professor Błaszczyk presents an informative discussion of the role of triangulation and proportion in the Euclid's study of area. I believe that footnotes 3 and 4 of the article misunderstand my paper, Baldwin, 2017. In footnote 3 he contradicts my assertion that Euclid's theory of area depends on proportion, by writing 'we have shown that Euclid's Theory of Area does not refer to proportions.' It should be clear from the first page of my paper that I regard Euclid's VI.I (area of a triangle is proportional to the base and 'height') as central to his theory of area. Professor Błaszczyk argues only that it is not needed for Books I and II. So his assertion that does not really contradict my paper; we refer to different texts by 'Euclid's theory of area'. He further argues that I assert that Euclid's theory of proportionality depends on the axiom of Archimedes although I nowhere demonstrate a logical dependence. This also seems to be a linguistic disagreement involving the meaning of 'depend'. My main objective was to publicize that the axioms Hilbert used in his actual geometrical results are first order. Hilbert uses the non-first order Axiom of Archimedes and Continuity axioms only for metamathematical purposes, including showing that the unique model of the theory with these axioms has exactly one model. As I thought was evident in my contrasting the proofs and conceptions of Hilbert and Euclid for VI.2 (if a parallel to the base cuts a triangle the sides of the resulting pair of triangles are proportional), I meant 'depends' as 'use' and Błaszczyk agrees that Archimedes is used. But Błaszczyk objects that I did not show that this dependence was essential. In fact the main point of my paper is that one can take the geometric propositions of Euclid at face value and validate VI.1 and VI.2 by Hilbert's entirely different method (interpreting a field and defining proportion from multiplication).

I did not assert that in any way this reflected Euclid's arguments but that it 'saved the data' from weaker hypotheses. It is of course perfectly consistent both that Blaszczyk has formalized Euclid's theory of proportion to show Proposition V.8 is logically dependent on the axiom of Archimedes and that a first order theory of Euclidean geometry (bi-interpretable with the theory of Euclidean field with the parallel postulate) proves VI.I.

I thank Błaszczyk for leading me to recognize a definite misstatement in my paper. Namely, the assertion that the proof of the Pythagorean theorem in I.47 depends on the theory of proportion. In fact, Hartshorne, 2000, page 203 shows that most of Book I, and in particular I.47, holds based on a theory Hartshorne calls equal content (similar in intent to Błaszczyk's triangulation). I was misled by the observation Hartshorne, 2000, Remark 23.6.1 that several of the later results in Book I depend on De Zolt's axiom (If a figure P is contained in a figure Q and their difference has non-empty interior, then they do not have equal content). In particular, the assertion 'halves of equals are equal' is needed to prove the converse of the Pythagorean theorem. Professor Błaszczyk believes this assertion is justified by Euclid's common notion V. Working with current notions of rigor, Hartshorne thinks further hypotheses should be made explicit. I think these are both reasonable standpoints.

References

Baldwin, J.: 2017, Axiomatizing changing conceptions of the geometric continuum I: Euclid and Hilbert., *Philosophia Mathematica* **a**, 32. online doi: 10.1093/philmat/nkx030.

Błaszczyk, P.: 2018, From Euclid's Elements to the methodology of mathematics. Two ways of viewing mathematical theory, *Annales Universitatis Paedagogicae Cracoviensis* **10**, 1–15.

Hartshorne, R.: 2000, *Geometry: Euclid and and Beyond*, Springer, New York.

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