FOLIA 430

Annales Universitatis Paedagogicae Cracoviensis

Studia ad Didacticam Mathematicae Pertinentia 17(2025)

ISSN 2080-9751
DOI 10.24917/20809751.17.7

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Leibniz's 1712 Review of Newton's *De Analysi*[*]

Abstract. We present a translation of Leibniz's review of Newton's *De Analysi* from *Acta Eruditorum* for February 1712 with a brief commentary.

1. Introduction

In the still ongoing debate about the primacy of either Newton or Leibniz in the invention of calculus, many sources have to be taken into consideration to properly assess the state of the art, even more so as already during these intellectuals' lifetimes, false testimonies have been given by both sides. One such source, which may provide not decisive, but significant contribution to our knowledge in this regard, is the Leibnizian review of Newton's *De Analysi per aequationes numero terminorum infinitas* [*On analysis by equations with an infinite number of terms*; usually shortened to *De Analysi*], published anonymously in the *Acta Eruditorum* for February 1712 (*Acta Eruditorum*, 1712, pp. 74–77).[1] In what follows, we provide the first, to the best of our knowledge, English translation of this piece.

The Latin text of the review is available in two sources: the first is its original place of publication, the *Acta Eruditorum*, the second is the second volume of *The Mathematical Papers of Isaac Newton*, edited by D.T. Whiteside (1968, pp. 259–262), where it is presented omitting one less significant paragraph.[2] The translation hereby presented renders the whole text as it appears in *Acta*.

De Analysi itself deals with the application of infinite series in geometrical analysis. We will not go into the details of the history of this text and its reprints, nor into its technical content, both of which the readers may find themselves in the

*2020 Mathematics Subject Classification: *01A45*

Keywords and phrases: *Newton, Leibniz, Calculus controversy, De Analysi, priority dispute, Acta Eruditorum*

[1] The *Acta Eruditorum* [*Acts of the Erudite*] was the first German scientific journal, published from 1682 to 1782. Among the people who supported its institution was Leibniz, who also contributed many articles over the years.

[2] As the edition seeks to "prune all which is not directly relevant to technical content" (Whiteside, 1968, p. 171).

edition of Whiteside (1968, pp. 165–171). It is to be known that Leibniz, visiting London in October 1676, was allowed to make a partial transcript of the text of *De Analysi*,[3] of which he wrote a review in 1711. Although Newton claimed so, there are no clear indications that Leibniz made any use of this copy and of the method of fluxions contained therein for his own purposes of developing calculus. The review, rather, shows Leibnizian interest in the specific formulation of Newton's method and in Newtonian notation, which he treats as inferior to his own notation of infinitely small quantities.[4] Although the review was published anonymously, the textual evidence shows beyond a doubt that its author was Leibniz – first, because he contributed to and authored numerous reviews written for *Acta*; second, because the text itself makes a reference to a previous review, undoubtedly authored by Leibniz, with words strongly suggesting a common author.[5]

Soon after print, the Leibnizian review received much more elaborate counter-observations written – in English – by Newton himself, which are also to be found in the edition of Whiteside (1968, pp. 263–273). Both texts show their authors' preferences for their own positions, but were also a chance to clearly define each one's position in reference to the notation and method of the other. In this regard, they also offer a glimpse into the intuitions which stood behind each type of notation.

The criticism of Newton – as one might expect – is directed at saying that Leibniz did not properly understand his method. While Leibniz is comparing both the method of 'first and last ratios' and that of 'infinitely small quantities' to the Archimedean method of exhaustion (which consists, in words of Leibniz, of 'showing that the error is smaller than any given quantity', see the Translation section in this article), Newton is renouncing such a view, stating firmly that '[...] in the method of first and last ratios the error is not proved to be less than any given ratio but to be none at all' (Whiteside, 1968, p. 271).

To this criticism of Leibniz, a more modern claim may be added. First, that Leibniz occupies himself only with an example of $z = \frac{2}{3}x^{3:2}$ and does not talk about a general solution $z = \frac{n}{m+n}x^{(m+n):n}$. Next, that he has not looked closely enough to notice a difference between how Newton uses his method in this particular example, to which Leibniz refers, and in the general example, which he does not mention at all and for the solution of which a different procedure is employed (see Błaszczyk, P. (2025), where two modes of deriving a formula for y in *De Analysi* are identified: one being correct from a modern point of view and the other not). As the texts of the reviews have not yet received a deserved attention from today's scholars of Newton and Leibniz, their content may still bring benefit to the study of the controversy dispute, which we hope to facilitate through this translation.

[3] The Latin text of this transcript is available in Whiteside as well (1968, pp. 248–259).

[4] *De Analysi* was written in 1669 by Newton as he wanted to claim the priority over the infinite series method. Thereafter, Newton has accused Leibniz of plagiarizing from a manuscript Leibniz has read during his stay in London. Among the mathematicians of the Royal Society, up till 1705 it was regarded that Leibniz has indeed copied his methods from the Newtonian script. The review hereby translated comes from a later period and was written by Leibniz after *De Analysi* was finally formally published in 1711. For a comprehensive description of the role of *De Analysi* in the Leibniz-Newton priority dispute, see Guicciardini (2009, pp. 372–384), while Osada (2022) is a recent voice in that debate.

[5] See Whiteside (1968, p. 259) and footnote 10.

Translation

Analysis per quantitatum series, fluxiones ac differentias cum enumeratione linearum tertii Ordinis.[6]

When William Jones, known for the publication of *Synopsis palmariorum Matheseos* (see *Acta* for the year 1707, p. 178),[7] has found in the papers of Mr. D. Collin's, among many other things sent to this man by the most celebrated mathematicians of his time, especially in Great Britain, some things that have been written by the great Isaac Newton, he thought about publishing them, and indeed he would not have done wrong if, by publishing either the entire correspondence of Collins or some longer extracts from it, he would have won himself the praise of the whole scholarly world. For since it is certain that this prominent man has corresponded extensively on scholarly matters, we do not have any doubts that his correspondence contains many great things that may bring benefit to both the study of the history of mathematics and the progress of mathematics itself. Nor is it an obstacle that a great part of these works may have already appeared in print elsewhere; for people versed in these things are aware of how beneficial it is to know when a great man has stumbled upon his findings. Nevertheless, when the prominent Jones observed that the Newtonian letters kept by Collins contained matters nearly identical with those the great Newton had already published, he changed his plans; however, obtaining the author's permission, he brought to light the treatise on the quadrature of curves, as he judged it to be written with both clarity and elegance and especially well-suited for instructing others, and appended to it some other analytical findings of Newton, on which we shall now speak.

Newton entitled this treatise on the quadrature of curves *On analysis by equations with an infinite number of terms* and in it he shows (see the figure below) that if $ax^{m:n} = y$, then the area is $\frac{an}{m+n}x^{m+n,:n}$. He does it in the following way: let $AB = x$, $BD = y$, the areas $ABD = z$, $\beta B = o$, $BK = v$, and the rectangle $B\beta HK$ be equal in area to $B\beta\delta D$. Let $\frac{2}{3}x^{3:2} = z$, or, equivalently, $\frac{4}{9}x^3 = z^2$, and by substituting $x + o$ for x and $z + ov$ for z, it follows that:

$$\frac{4}{9}inx^3 + 3x^2o + 3xo^2 + o^3 = z^2 + 2zov + o^2v^2.$$

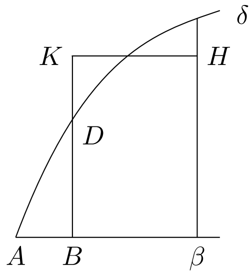
Then, with the equal terms $\frac{4}{9}x^3$ and z^2 removed and the rest divided by o, what remains is:

$$\frac{4}{9}in3x^2 + 3xo + o^2 = 2zv + ov^2.$$

If it is now assumed that o stands for nothing, v and y will be equal, and the terms multiplied by o will disappear. Hence, what remains is $3x^2.\frac{4}{9} = 2zv$ or $x^{1:2} = y$. Therefore, by converse, if $x^{1:2} = y$, then $\frac{2}{3}x^{3:2} = z$. Newton applies this method

[6] Such was the title of William Jones' 1711 collection of Newton's mathematical works.

[7] See *Acta Eruditorum* (1707, pp. 178–181). The reference is to the review of the work of Jones printed in *Acta*. As Whiteside notes (1968, p. 259), although this review has not been formally authored by Leibniz, he may have contributed significantly to it, as it was the case with many reviews written for *Acta* at that time.



first to simple curves, in case of which the value of the y itself is a single term, for example, when $4\sqrt{x} = y$ or $1 : x^2 = y$; then to composite ones, in which, for example, $x^2 + x^{3:2} = y$, or $x^2 + x^{-2} = y$. Finally, he expands the value of the y itself into a series, either by division, following the example of Mercator, as in $a^2 : (b + x)$, or by taking the roots, as in $\sqrt{a^2 + x^2} = y^2$. On this occasion, he also presents the method of taking roots of both simple and affected equations,[8] which had been already presented in the *Algebra* by Wallis, who took it from Newton's letters to Oldenburg. In some examples, it is also told how to apply this method to finding lengths of curves and quadratures of mechanical curves.[9]

Attached to this treatise are excerpts from Newton's letters sent to Oldenburg and published by Wallis in the third volume of his works; besides these, others were also sent to Wallis and printed in his *Algebra*, as well as a fragment of a letter sent to Collin's, dated November 8th, 1676.

The most illustrious editor once caused two treatises, attached to the *Optics* of Newton, to be reprinted: one of them deals with the quadrature of curves, the other enumerates third-order curves. Both were discussed in the *Acta* (1705, p. 30 and further).[10] Yet the most illustrious editor would have rendered himself of outstanding merit to geometers if he had at once included the proof of the number of third-order curves, which Newton, if asked, would certainly not have refused. Indeed, he would render a great service even now, should he publish it as a sort of appendix or on some other occasion.

Lastly, there exists a certain very short treatise to which the title *Differential method* is given in a very special sense, and which was transcribed from the autograph of Newton by the most illustrious editor himself. It includes a method of describing curves from given differences between the differences of the coordinates and rests on the problem of drawing a parabolic curve through any number of given points. He shows that, since one can take the quadrature of every parabolic curve, one can also get an approximate quadrature of every curvilinear figure as long as some of its coordinates can be found. Nothing else is required here besides that through the terms of given coordinates a parabolic curve can be drawn.

Besides, as the most distinguished editor prefers the method of first and last

[8] I.e., polynomial equations in two variables.

[9] I.e., curves that can be generated by motion, e.g., a cycloid.

[10] See *Acta Eruditorum* (1705, pp. 30–36). The reference to this work, as beyond a doubt one written by Leibniz, provides a firm ground for claiming the authorship of Leibniz for the translated review – consult (Whiteside, 1968, p. 259).

ratios to the method of infinitely small quantities, it is to be noted that they differ only in the manner of expression and, when it comes to a rigorous demonstration, each should be referred to the Archimedean method, so that it is shown that the error is smaller than any given quantity. And as in the preceding calculations o and ov are employed, who does not see that, in truth, what is used are the infinitely small quantities, with o standing for $\mathrm{d}x$ and ov for $\mathrm{d}z$? It is known that Fermat and others have already employed o in such cases. But with the invention of differential calculus by the great Leibniz, it is no longer the case that simply "nothing" is used as a quantity; instead, certain special vanishing quantities are introduced, namely such that express the rate at which the primary quantities are decreasing to nothing. In this way, $\mathrm{d}x$ or $\mathrm{d}z$ are special quantities tied to x and z, being a kind of modification of x and z themselves, namely the difference between two x's or two y's, but tending to zero. And so the quantities whose modifications are required to express the curves are not multiplied; hence, even the equations of transcendental curves[11] are given via the sole relations between ordinates and abscissas. Our Leibniz, following the examples of both Cavalleri and Robervalli, sometimes used this way of expression, sometimes the other – that is, sometimes the infinitesimals, sometimes the motion, also called the continuous transit or "fluxion" – as it was most convenient for him. He extended the use of this "transit" beyond geometry to physics itself, as he discovered a certain new principle called the *law of continuity*, by means of which many erroneous things in physics can be straightforwardly disproved. He proposed this principle many years ago in *Novellis Reipublicae Litterariae* of Pierre Bayle[12] and illustrated it with examples.

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[11] I.e., curves that cannot be defined using polynomial equations.

[12] The *Nouvelles de la république des lettres* [*News from the Republic of Letters*] was a book review journal published in the years 1684–1718 and written primarily by Pierre Bayle (hence the adjective *Baelianis* used by Leibniz in the original).

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