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What Teachers Learn From Students' Problem Posing in Algebra: A Self-Study With Eighth Graders*

Abstract. This self-study explores how reflective analysis of eighth-grade students' mathematical problem-posing and problem-solving activities enhances teachers' instructional insights and influences pedagogical decisions. Implemented within a design-based research framework, the study examined 79 Hungarian eighth graders' abilities to create structurally and semantically correct algebraic word problems. Results indicated that while most students successfully generated structurally coherent problems, many encountered difficulties formulating semantically meaningful contexts. Reflective analysis highlighted specific instructional gaps, underlining the importance of explicit teaching interventions targeting semantic comprehension. Overall, carefully thinking about how students create problems proved to be a useful way to identify areas for improvement in teaching, helping to make better adjustments that support students' understanding of concepts and their ability to think mathematically.

1. Introduction

Assessing the impact of pedagogical interventions is problematic; classroom interventions are often so complex that it is difficult to conduct reliable quantitative studies, and qualitative methods tend to come to the fore (Brown, 1992). The author of this article is a practicing teacher who, with the help of a university expert, conducted a design experiment, initially in her class and school and later involving several schools. In this paper, following a self-study approach, we report on how we used the problem-posing method to assess student understanding in a design experiment to promote meaningful algebra learning. Self-study, as characterized

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by Schuck (2002), emphasizes rigorous reflection on personal practice not only to improve one's own teaching but also to generate broader insights valuable to the teaching community. Specifically, our self-study examined the effectiveness of a problem-posing approach for assessing student understanding in algebra, positing successful problem posing as a possible indicator of meaningful learning. The idea itself is not new, as one of the characteristics of problem posing is that it provides the teacher with a "window into the student's mind" (Silver, 1994), but we refined this general principle by examining student understanding from both structural and semantic perspectives. Structural understanding refers to understanding the structure of a mathematical model, whereas semantic understanding refers to the relationship between the mathematical model and the real world.

Mathematical problem posing is a pedagogical strategy that promotes active learning and has gained increasing importance internationally, in line with educational trends that emphasize the acquisition of deep and meaningful mathematical knowledge by students (Cai and Hwang, 2022; Silver, 1994). In Hungary, problem posing is part of the mathematics teaching tradition, supporting students' active participation in learning (Kovács, 2022). Problem posing enables students not only to better understand mathematical concepts but also to connect them meaningfully to real-life experiences. Consequently, it promotes positive attitudes and motivation towards mathematics (Báró, 2022; Bevan and Capraro, 2021). Our previous research with sixth-grade students showed promising results in the classroom application of structured problem-posing tasks, particularly in numeracy skills (Kovács et al., 2023). Building on previous experiences and results, the present study examines the effectiveness of a similar approach with eighth-grade students in algebra learning, using the "sense-making algebra" approach. This approach emphasizes conceptual understanding in contrast to rote learning. The present study examined the structural and semantic dimensions of problems created by eighth-grade students. Structural understanding involves recognizing mathematical schemas or patterns in problems, whereas semantic understanding refers to students' ability to connect meaningfully mathematical problems to real-world context (Briars and Larkin, 1984; Riley et al., 1983). Examining these two aspects provides more profound insight into students' comprehension difficulties and their ability to transfer learned problem structures to new contexts. It also provides a novelty for our research. Therefore, our research question is: How does systematic reflection on students' problem-posing processes inform teachers' understanding of their instructional practices and pedagogical decisions? Specifically, we examine how students' mathematical proficiency influences their ability to generate structurally and semantically correct problems and how problem posing can be a diagnostic tool for teachers to identify students' specific difficulties. The results indicate that while most students successfully generated structurally coherent problems based on a model problem, a significant proportion encountered difficulties in creating semantically valid problems, highlighting the critical role of semantic understanding in capable mathematical problem posing. These findings underscore the importance of explicitly targeted pedagogical interventions that address semantic comprehension in mathematics instruction in classrooms.

2. Literature background

2.1. Problem-posing

The broad range of definitions, conceptualizations, and implementations of problem posing in mathematics education research demonstrates that the field of problem posing is still very diverse and lacks definitions and structures (Papadopoulos et al., 2021). In his famous four-step model of problem solving (understand the problem, devise a plan, carry out the plan, look back and reflect), Polya (1945) considered problem posing as part of problem solving. In the second step of the problem-solving process (developing a plan), he proposed posing several questions that encouraged the student to prepare a supporting task, such as a more general or specific one. Furthermore, in the look-back phase, he encourages the learner to apply the achieved result to other problems. However, most of the efforts concerning the conceptualization of problem posing go back to Kilpatrick (1987) and Silver (1994). These scholars relied on several works of Polya (1945, 1954, 1981), but they defined problem posing as a separate field in mathematics education research. Silver's (1994, p. 19) broad definition states that "Problem posing refers to both the generation of new problems and the re-formulation of given problems. Thus, posing can occur before, during, or after the solution of a problem." In this paper, the focus is on "re-formulation" of a given problem; to be more precise, a student is expected to reformulate a problem on a well-elaborated model problem (Ellerton, 2013), mainly changing the word problem situation, for example, inserting a personalized environment or changing the numerical data.

Several studies have examined the advantages and positive effects of problem posing in mathematics education. Cifarelli and Sevim (2015) summarize why problem posing is helpful in problem solving in three ways: (1) It helps students broaden the original problem's perspective and the scope of the problem. (2) The problem-posing activity, which can lead to better conceptual understanding, highlights the importance of this activity for learners. (3) Learners actively monitor and evaluate the usefulness of their ideas when they begin problem posing and actively raise new questions and problems when necessary. Other authors emphasize that group discussions of problems created by students allowed students to reflect on different problems and explore new possibilities, such as open-ended tasks (Bonotto and Santo, 2015). Classroom discussions of these problems helped students' critical thinking because they felt free to explore and solve problems of their own creation (Bonotto, 2013). Several studies have articulated the motivational role of problem posing in classrooms (Báró, 2022). Bevan and Capraro (2021) found that the freedom to choose the context of problems increased students' interest and autonomy in posing and solving problems. Such problem-posing activities can increase students' interest in mathematics and their confidence in their mathematical abilities. Bonotto and Santo (2015) argued that problem posing can foster creative thinking because learners can generate original and even open-ended problems.

In this paper, we also utilize an approach that highlights the diagnostic potential of student problem posing as a tool for teachers to gain deeper insight into students' mathematical understanding. Several scholars have emphasized this aspect of problem posing. Silver (1994) considered problem posing as a window for learners' mathematical understanding. He also argues that problem posing is not only a window, but also a mirror that reveals the content and nature of learners' mathematical experiences. Ellerton (1986) concluded in her research on primary school students that there is a correlation between students' general mathematics performance and the difficulty level of the problems they formulated. Pupils with higher abilities created problems that required more complex calculations or multiple operations than their less able peers. Higher-ability pupils also performed better at solving the problems they created. Ellerton suggested that the problem-posing method is a valuable diagnostic tool for studying gifted pupils, for whom routine tasks are usually completed quickly and accurately. Thus, examining student-generated problems allows educators to diagnose specific conceptual misunderstandings and procedural difficulties, making this method particularly valuable in assessing student learning in mathematics.

2.2. Sense-making algebra

Problem-posing can become a tool for learning algebra and play a decisive role in developing students' algebraic thinking. It allows students to discover and explore the algebraic structures inherent in arithmetic situations (Cai and Hwang, 2022). The concept of sense-making algebra emphasizes this kind of cognitive engagement, enabling learners to actively connect and generalize mathematical ideas.

Sense-making algebra is an approach to learning and teaching algebra that focuses on conceptual understanding, rather than memorizing or implementing procedures. This approach includes several key elements that reflect the main components of mathematical reasoning. Palatnik and Koichu (2017), referencing (Kieran, 2007), state that sense-making in an algebraic context involves a series of generational, transformational, and meta-level activities. Generational activities involve creating and interpreting algebraic objects (e.g., expressions, equations) representing observed phenomena or patterns. Kieran (2007) argued that such activities help students understand the significance and purpose of algebraic expressions, fostering recognition of algebra as a language. Transformational activities involve changing and working with algebraic objects to help students become skilled at procedures and understand important ideas like equivalence. Finally, algebra as a meta-level activity refers to mathematical activity for which algebra is used as a tool. These activities, including problem solving, modeling, and problem posing, provide the context, sense of purpose, and motivation for engaging in the generational and transformational activities. In this sense-making process, learners formulate and justify statements, generalize, find the mechanisms behind algebraic objects, i.e., gain a more profound understanding of the basic mathematical principles and logical relationships behind formulas, and can connect and unify the mathematical representations they are studying, such as formulas, diagrams, or numerical sequences.

The main characteristics of the sense-making algebra approach to teaching that are relevant to our classroom intervention are as follows.

Learning in context. Pupils engage in introductory algebra through real-life problems, which helps them become aware of algebra's importance. Knowledge is not merely abstract information; it is shaped by concrete situations and experiences (Brown et al., 1989). Problem posing is considered an important means of creating livable situations (Stoyanova and Ellerton, 1996).

Utilizing multiple representations. This approach encourages using different forms of representation, such as natural language, graphical, numerical, and symbolic representations. This flexibility allows learners to approach problems from different perspectives and to interpret problems intuitively (Janvier, 1987).

Formulating generalizations. Formulate generalizations and patterns from concrete examples, a key aspect of algebraic thinking. This generalization can occur without formal algebraic notation (Schoenfeld, 1992).

Developing mathematical reasoning. Although reasoning has often been identified with formal proofs, it is a way of thinking and a set of skills that should be an essential part of all mathematical activity. The development of reasoning occurs through several stages. The first stage is empirical reasoning, which leads through intuitive explanations (pre-formal reasoning) to formal proofs based on logical principles. Each level is equally important, so learners need sufficient time to develop and consolidate their knowledge before moving on to a higher level. Moreover, learners may be able to switch between levels of reasoning, even within the same context (Keazer and Menon, 2015).

Overall, the sense-making approach to algebra learning aims to enable students to experience learning algebra as an active, useful, and meaningful human activity. To effectively integrate sense-making algebra in classrooms, structured teaching frameworks that promote active learning and problem posing, such as Ellerton's Active Learning Framework (Ellerton, 2013), are especially beneficial.

2.3. Active Learning Framework for problem posing

We introduced the problem-posing learning method in the classroom based on Ellerton's (2013) Active Learning Framework (ALF). One of its characteristics is that student activity gradually increases during the lesson, while teacher guidance is reduced and transformed accordingly. Ellerton organizes the ALF classroom activities into six phases; however, in our research, we adapted this model into four main phases, as summarized in Table 1. In the first step, the teacher presents a model problem, which is elaborated on in detail in a whole-class activity. At this stage, the pupils primarily observe, memorize, and remain receptive. This is followed by a practice activity in which the pupils' activity increases, as they use and repeat the steps observed while elaborating on the model problem. After completing the practice tasks, learners are asked to pose problems based on the same structure as the model problem. At this stage, the learners' activity increases further as they reflect on what they have learned and experiment. Here, teacher guidance diminishes and changes into a supportive, supervisory role, mainly supervising and monitoring the classroom activities. Finally, in the problem-solving phase, the teacher selects problems created by the pupils that are suitable for class discussion and collaborative solving.

In the original framework, the introductory part before the problem-solving phase was divided into three parts: the teacher models examples, draws attention to the textbook, and the students locate examples. Based on findings from a pre-

Model problem	Problem solving	Problem posing	"My classmate's problem:" Problem solving
Processing new knowledge, the teacher models examples, and draws attention to the textbook	Students look for examples and solve problems based on the model.	Students create a problem with the same structure as the model.	The class discusses and solves problems created by the pupils.

Table 1: ALF classroom activities

vious action research study (Kovács et al., 2023), we adapted this framework into four phases because it more naturally aligns with Hungarian teachers' traditional lesson plans (introduction of new material, guided practice, independent student work, and concluding class discussion). Our action research demonstrated that this adjusted four-phase model integrates easily into existing classroom routines and timing structures, making it especially practical for classroom teachers who typically follow a structured, instructional approach. Furthermore, as Cai et al. (2015) consider, problem posing is a valuable closure activity that allows learners to consolidate and think critically about what they have learned.

Based on the positive experience of our previous research (Kovács et al., 2023), we concluded that the ALF method can be effectively integrated into the sense-making algebra approach. Our former findings demonstrated that sixth-graders (aged 11–12), even without prior experience in problem posing, successfully engaged in this activity when structured according to the adapted four-phase ALF model. Furthermore, the study showed that the success of students in problem posing was closely related to their general mathematical abilities, with students achieving lower grades in mathematics typically experiencing greater difficulty. These findings underline the importance of structured guidance, especially for lower-achieving students, to help them engage meaningfully with algebra through problem posing. Thus, the structured approach of ALF not only supports effective classroom organization but also directly facilitates the deep, conceptual engagement required by sense-making algebra.

3. Analytical framework

In this study, we apply structural and semantic comprehension perspectives to analyze mathematical problems created by students. To interpret these concepts, we rely on Riley et al. (1983) and Briars and Larkin (1984). Structural understanding means recognizing the problem's form, schema, or pattern, and the arrangement of information in the text, recognizing the relations between different parts. Semantic understanding involves comprehending the real-world meaning, the purpose of the problem, and the implications of the problem's content. In the present study, these notions were operationalized in relation to students' posed collaborative-work problems. A problem was treated as structurally correct if it included at least two actors and sufficient independent performance data to determine one unknown quantity, and as semantically correct if the context allowed these performances to be meaningfully combined in a realistic way, as in cleaning or filling a pool, but not in activities such as writing poetry or making soup.

Students with good structural understanding might recognize that a situation fits a known schema, but might misunderstand the real-world meaning if they lack semantic understanding. Conversely, semantic understanding allows grasping the context, but without structural recognition. The student might struggle to choose the appropriate operations or schemes for solving the problem. In our analysis, we pay special attention to the interactions between structural and semantic understanding, as students' challenges often arise from discrepancies or mismatches between these two types of comprehension. Although the cited frameworks were developed for arithmetic-based word problems, the basic concept fits well with our research.

The problem-forming process ends when a person solves their problem (or that of a classmate) and considers whether the solution produces a meaningful result in the given context. Thus, in the case of structurally well-formulated problems, the evaluation of solutions cannot be omitted. Following the ALF framework, this learning phase is more about solving routine tasks, as students have already completed the practice phase. Solving routine tasks uses mental schemas that identify the type of task and the appropriate algorithm. This requires some cognitive abilities: the ability to recognize similarities and the ability to imitate algorithms (Vinner, 1997). The problem-solving strategy is chosen based on the similarities between the task and tasks with known solution algorithms, based on structural properties. In this study, we evaluate the selection of the appropriate algorithm and its correct implementation when evaluating solutions. Our goal is not to classify student errors but to identify student difficulties to gain insight into mastering algebraic concepts. As highlighted by Säfström et al. (2024), characterizing students' specific reasoning difficulties during problem solving by focusing on the phases where students get stuck supports targeted diagnostic and instructional efforts.

4. Method

4.1. Design-based research

The research presented in this article is part of a design-based research study (Kelly, 2003) among eighth graders on word problems solvable with linear equations. The pedagogical problem we aim to address is that students find it challenging to solve word problems with equations at the beginning of their algebra studies. Previous research also emphasizes this phenomenon (Bush and Karp, 2013; Jupri and Drijvers, 2016; MacGregor and Stacey, 1998). The intervention addressed this problem by using different representations in a problem-based learning environment (Kónya and Kovács, 2022), including problem posing. In other words, during the experimental lessons, students learned alternative ways to solve word problems without explicitly relying on equations. Additionally, various representations were introduced to help students formulate equations effectively. Problem posing activities were also incorporated as integral parts of the learning process.

Design-based research stems from the recognition that in classroom-based pedagogical research, it is virtually impossible to single out a single factor (intervention) and measure and evaluate its impact (Brown, 1992). By design-based research, we mean the design and control of an innovative learning environment (reminiscent of engineering methods) in which innovation is experimentally studied within an authentic classroom environment (Swan, 2014). It orchestrates a slice of daily life, involving several interrelated factors. This research paradigm involves designing, developing, and refining a product or process through implementation, observation, analysis, and redesign cycles.

The main phases of design-based research (analysis and exploration, design and implementation, evaluation, and reflection (McKenney and Reeves, 2012) are cyclical, and development is iterative rather than linear. For example, the evaluation and reflection of one cycle impact the next design phase. In educational interventions, it is typical that the initial cycle involves a preliminary design implemented under optimal conditions with considerable researcher guidance and control. In subsequent cycles, however, the intervention is tested under more realistic classroom conditions, often by teachers who were not involved in the original design, thereby evaluating its practicality and effectiveness in authentic educational contexts.

4.2. Participants and tasks

This research is a continuation of our previous work reported in Lócska et al. (2023). The research was conducted in 2021–2022 and 2022–2023. In total, 79 eighth-grade students participated in two Hungarian schools characterized by mixed-ability classrooms, under the guidance of four teachers. Students are referred to as S01–S79 as follows. In the first cycle, the teacher-researcher, i.e., author of this paper, directly controlled the experiment; however, in the second cycle, she provided only lesson plans for teachers in the research, with the freedom in implementation.

The lessons followed the ALF paradigm, beginning with a teacher-guided model problem solution, followed by structured practice activities, and culminating in autonomous problem posing and solving tasks. When designing the lesson, an important aspect of the ALF was the gradual increase in student activity. While the model problem solving was strongly guided by the teacher, the problem posing and solving were autonomous tasks.

This paper focuses on the creation and solution of "collaborative work" type text problems. These problems involve at least two living (e.g., persons as in Examples 1 or 2 in Table 2) or non-living (e.g., taps as in Example 3 in Table 2) actors performing tasks together in various contexts. The actors work with a consistent performance, so that if several actors work on the same task, they finish sooner. When designing the lesson, the practice tasks that preceded the students' independent problem posing were designed to be consistent with the types of tasks in the textbooks. On this basis, the three tasks covered three contexts, see Table 2.

	Task	Context
1.	Bob keeps goats and Kevin sheep on the farm. A hay cart lasts 3 weeks for the sheep and 6 weeks for the goats. Bob and Kevin agree to share a cartload of hay. How many days will the feed last?	Shared meals
2.	Stuart, Bob, and Kevin are going to clean up the castle they built together because they want to watch the football match on TV in nice conditions. Kevin alone would take 3 hours to clean the castle, Stuart would do the same, and Bob would take 2 hours. The match starts in 55 minutes. Will they clean up by then?	Cleaning
3.	The swimming pool takes 3 hours to fill up through one tap and an hour and a half through the other. How long will it take to fill the pool if both taps are opened?	Filling the pool

Table 2: Example tasks used in the practice phase

4.3. Categorization of students' work

Students' work was coded using the analytical framework developed for this study. Each posed problem was first classified along two dimensions: structural correctness and semantic correctness. Structurally, a correct problem involves at least two actors and explicitly provides sufficient independent data points on their performance (such as the time required individually or collectively) to calculate a single unknown quantity. For example, for two actors, two independent data points are required; for three or more actors, additional data must be provided accordingly to ensure solvability. Semantically, a problem is correct if the activity described is realistically suitable for collaborative work. This means that, when performed together, it can be completed in less time than when performed individually, while each actor's performance rate remains consistent. Counterexamples include intellectual activities (such as writing poetry, learning, playing the piano, and most sporting activities). For structurally correct problems, the accompanying solutions were additionally coded into one of three categories: stuck on strategy selection, proper strategy but wrong implementation, or proper strategy and correct implementation. The author and a university expert coded the students' work independently using these predefined criteria. They then compared their classifications and resolved discrepant cases through discussion until consensus was reached.

In the following, we give illustrative examples for all four possible categories in the first phase of coding. Student S34 provided a suitable example of a *structurally and semantically correct* problem:

> Aaron alone cleans the house in 3 hours, while his sister, Eniko, takes 2 hours. How long does it take them to clean the house together?

In this example, cleaning is a realistic collaborative activity, and we used it as a model task with three actors. The two individuals' performances can be meaningfully combined, reducing the total completion time. S41's problem illustrates a *structurally correct, but semantically incorrect* solution.

> Kevin makes soup in 1 hour, Stuart takes 1 hour 10 minutes, and Bob takes 1 hour 30 minutes. How many hours does it take three people to make the same soup?

All the data for calculating individual performances can be found, but cooking is not an activity where performances can be added together.

The last category is the *structurally incorrect, but semantically correct.* The next student chose to fill a basket while picking bananas.

> Bob, Stuart, and Kevin were collecting bananas for their 5-liter basket. In one hour, Bob collected 12 bananas and Stuart 22. How many bananas did Kevin collect if the basket only holds 116 and he collected less than 40%? (S76)

The example is semantically correct and resembles the "filling the pool" task. The structural formulation starts well: the basket can hold 116 bananas, and the characters' performance is specified. However, the question does not fit the task type and is essentially a simple arithmetical task regardless of the actors' performance.

A *structurally and semantically incorrect* example is the work of student S66

> At a party, there were two candy dispensers. One is good for 2 hours, but the other is good for 6 hours. How long is the second candy dispenser good for? (S66)

This student provides data in the text that do not refer to the actors' performance (i.e., the work done per unit of time), but instead to durability or availability, which are not combinable in a meaningful way. Perhaps the student was influenced by the "filling the pool" problem. Moreover, the question explicitly asks for information already provided in the text ("the second dispenser is good for 6 hours").

After classifying students' posed problems along the dimensions of structural and semantic correctness, we turned to the second phase of the analysis: evaluating the accompanying solutions of structurally correct problems. This second step was necessary because a well-posed problem does not in itself guarantee successful problem solving. Drawing on the reasoning-focused diagnostic approach suggested by Säfström et al. (2024), we pay attention to identifying students' difficulties at distinct phases of the solution process. Specifically, we categorized solutions into three groups to identify where reasoning difficulties emerge. These three groups are: (1) stuck on strategy selection, (2) proper strategy but wrong implementation, and (3) proper strategy and correct implementation. The details of each category are explained below.

Stuck on strategy selection. The student either does not start the solution or selects an inappropriate strategy. The student may begin a strategic approach, but fails to reach a point where it can be executed procedurally.

S19's work is structurally and semantically correct.

> Uncle Józsi fills the same ice cream van in 2 hours, while Ferenc fills it in 30 minutes. How many seconds would it take them to fill the van if they worked together?

The student was stuck in the problem-solving process (Figure 1). They start filling in the table correctly with the data he has been given, showing how long it takes to complete the job alone. They also make a correct conversion, 30 minutes = 0.5 hours. However, when writing equivalent fractions by multiplying the numerator and denominator by two, they interpret it as the work done in two hours. Their initial step is only an algebraic expression ($\frac{x}{2} + \frac{x}{0.5}$), and not an equation, pointing clearly that the student has difficulty with the essence of the equation that leads to the answer. The last step, following the algebraic expression above, is $2 + 0.5 = x$, reflects a procedural shortage.

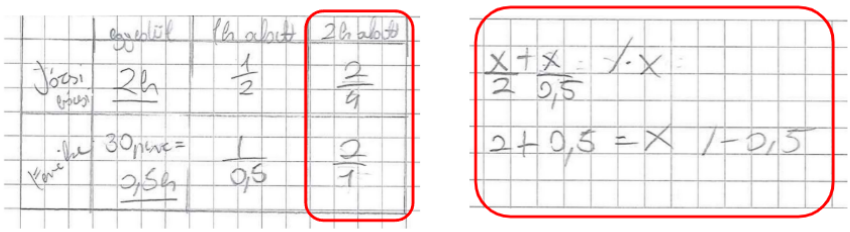

Figure 1: S19's problem-solving performance and translation of their table

	alone	1 hour	2 hours
Uncle Józsi	2 hours	$\frac{1}{2}$ part	$\frac{2}{4}$ part
Ferenc	30 minutes = 0.5 hours	$\frac{1}{0.5}$ part	$\frac{2}{1}$ part

Proper strategy but wrong implementation. The student correctly identifies the appropriate structural schema and selects a suitable strategy; however, they make errors during procedural execution (e.g., algebraic transformations, arithmetic operations) or fail to complete the procedure.

The problem text prepared by student S53 is structurally and semantically correct.

> A pool is filled with one tap in 4 hours and the other in 2 hours. How long will it take to fill if both taps are open? (S53)

This student tried two strategies: solving the problem with an equation, and also tried solving the problem through graphical representation (Figure 2). Both strategies are applicable to the problem. Moreover, the student executed the initial steps correctly. Ultimately, they tried to solve the problem with an algebraic solution. The student organized the data in a table and correctly wrote the necessary equation. However, in the last step, they made a procedural error (see the red box in the figure).

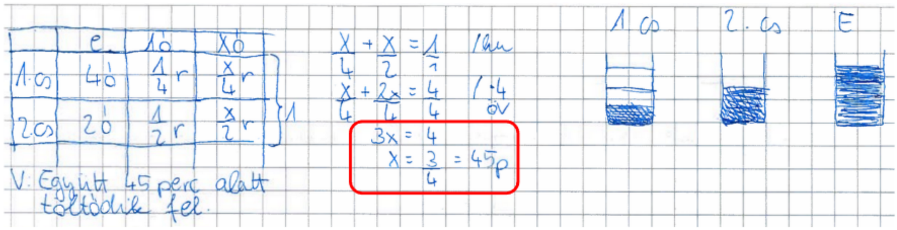

Figure 2: S53's problem-solving performance and translation of their table

	alone	1 hour	x hour
1. tap	4 hour	$\frac{1}{4}$ part	$\frac{x}{4}$ part
2. tap	2 hour	$\frac{1}{2}$ part	$\frac{x}{2}$ part

Note. The translation of the text in the bottom-left corner is: Together, it takes 45 minutes to fill [the pool].

Proper strategy and correct implementation. The student correctly recognizes the structural schema and executes the necessary procedural steps without errors, demonstrating robust comprehension.

This categorization allows us to diagnose more deeply the specific reasoning challenges and comprehension difficulties students face, ultimately informing targeted instructional strategies.

5. Results and discussion

5.1. Problem-posing activity

All the categories defined in the previous section were present in the texts of the problems created by the students (Table 3). There were structurally and also semantically incorrect, structurally correct but semantically incorrect, and structurally incorrect but semantically correct items, as well as problems that were correct in every respect.

		Structure		Total
		Correct	Incorrect	
Semantics	Correct	61	1	62
	Incorrect	8	9	17
Total		69	10	79

Table 3: Frequency of problem types proposed by students

One key finding emerging from the self-study reflection is the critical pedagogical insight from systematically analysing students' posed problems. Teachers may recognize specific instructional gaps and opportunities for refinement by engaging deeply with structurally and semantically correct and incorrect student problems. Of the 79 students, 69 produced structurally correct problems. These students understood the structure of the problem, i.e., the mathematical model, from a procedural perspective. The semantic analysis of the task yielded weaker results; 17 students could not connect the model to a meaningful everyday context. This gap highlights the need for teachers to explicitly highlight and discuss the boundaries and conditions for realistic applications of mathematical models during classroom instruction.

In the experiment, two eighth-grade classes (40 students altogether) had previously participated in research on problem posing in sixth grade (Kovács et al., 2023). These students were familiar with the problem-posing activity. The result showed no correlation between previous experience and the correctness of the created problems (Fisher's exact test: $p = 1$, see Table 4). This experience confirmed the findings of previous research, which suggested that problem posing can be used confidently in the classroom with the modified ALF model.

	Structurally correct	Structurally incorrect	Total
There is previous experience	35	5	40
No previous experience	34	5	39
Total	69	10	79

Table 4: Students' problem posing in terms of previous experience

5.2. Problem-solving activity

We also examined the problem-solving performance of students who had posed structurally correct problems (Table 5).

Category	Number of students
Stuck on strategy selection	30
Appropriate strategy but wrong implementation	9
Appropriate strategy and correct implementation	30
Total	69

Table 5: Students' problem-solving performance

The evaluation of problem-solving performance also offered significant reflective insights. The varied success in students' solutions highlighted an essential self-study revelation: even structurally and semantically correct problem formulations do not necessarily imply fluent problem-solving abilities. Students frequently encountered difficulties in strategy selection and procedural execution. The teacher reflected on these difficulties and recognized the need for differentiated instructional support and strategies explicitly targeting these problem-solving phases.

Reflecting on the following problem also highlights the difficulties of effective classroom implementation. S07 gives an example of a structurally and semantically correct problem:

> The pool can be filled with three taps. One tap takes 5 hours to fill, and the other takes 6 hours to fill independently. The three together take 3 hours. How long does it take the third tap alone to fill?

In the quoted example, there are three actors; two performance data and the combined work data are given, and the question is the third performance data. However, this example shows that simply stating the problem is not enough, even if it is structurally and semantically correct, because the two taps alone would overflow the pool in three hours. Thus, solving and reflecting on the problem is part of problem-solving and, therefore, the learning process.

This example also highlights that problem posing and problem solving must be viewed as a unified whole, as solving the created problem is an important part of the learning process. At the same time, this leads to pedagogical difficulty in problem-posing: How can teachers manage the problems created by students in the classroom? As Kovács and Kónya (2021) pointed out, this is one of the antinomies of problem-posing method in the classroom: we want as many problems as possible to be created by students, but the more problems there are, the more difficult it is to manage them in the classroom and decide whether to include them in the lesson, reject them, or postpone them. This problem shows that incorporating it into the learning process would have been instructive, but this did not happen, primarily because the student did not provide a solution. At the same time, solving this problem also leads to a cognitive conflict, which is not always effective as a pedagogical tool: it can be successful with more capable students, but counterproductive with less capable ones (Zohar and Aharon-Kravetsky, 2005).

6. Summary and pedagogical implications

In this self-study, we systematically explored how reflective analysis of eighth-grade students' problem-posing and problem-solving activities can enhance the teacher's understanding of instructional practices.

The results indicated that while most students could create structurally correct problems, many encountered difficulties in semantic accuracy and realistic contextualization. Reflective analysis revealed that the semantic dimension of problem posing requires explicit instructional attention, beyond mere structural understanding. Teachers should therefore proactively integrate discussions of realistic and unrealistic contexts within problem-posing tasks, explicitly addressing the limitations of mathematical models and their real-world applicability. The detailed categorization of students' difficulties during problem solving further highlighted the necessity for targeted instructional differentiation. Students need explicit support at different phases of the problem-solving process, particularly strategy selection and procedural implementation.

Additionally, the correctness of students' posed problems did not depend on previous experience. The work of students who had previously engaged in problem solving on several occasions showed no significant difference compared with the work of other students. So, problem posing can be introduced into lessons at any time.

From a self-study perspective, the key implication is that teacher reflection on student-created problems offers a profound diagnostic tool that uncovers hidden instructional gaps and provides concrete directions for pedagogical improvement. Future instruction should systematically integrate reflective discussions, contrasting structurally and semantically appropriate and inappropriate examples, thus fostering students' deeper conceptual understanding and supporting their mathematical reasoning.

Our study confirmed that by examining the problems that students create, the teacher can gain deeper insights into students' understanding (Silver, 1994). Subtleties that would not be revealed in a traditional problem-solving activity can be revealed. Moreover, the relationship between problem posing and problem solving is complex. Structurally and semantically correct problem creators will not necessarily be successful in solving the problem.

7. Limitations and further research

The study has limitations, including potential biases due to the self-study method, a small sample size of 79 students, and a focus on structural and semantic correctness in algebraic word problems, omitting some other factors, such as metacognitive aspects. Future research could include additional observers to enhance objectivity and address Hungary's cultural and educational context.

One of the characteristics of design-based research is its cyclicality (Swan, 2014). We plan to extend the research presented in this paper by adding a further cycle, thus broadening the range of participants and using the reflection on the previous cycles.

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