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Modern Mathematics/New Math: From Its Heights in the 1960s to Its Fall in the Early 1970s*

Abstract. After the Royaumont Seminar in 1959, modern mathematics/New Math conquered the whole world. However, its introduction during the 1960s varied among countries, determined in part by local history, culture and traditions, educational systems, and the influence of national reformers. We discuss exemplarily how modern mathematics was implemented in a number of countries or parts of the world. From the early 1970s, the decline was initiated. Criticism sounded louder and louder; in some countries it even led to a real Math War. In most countries, modern mathematics/New Math did not survive the 1970s and the retrospective assessment was typically (very) negative; *Time Magazine* included New Math in a list of the 100 worst ideas of the century. However, from its ashes arose substantial international cooperation in the field of mathematics education, and this field gradually acquired the status of an autonomous scientific discipline.

1. Introduction

The 1959 Royaumont Seminar was a crucial gathering for the modern mathematics/New Math movement for at least three reasons: (1) it was the first time in history that European mathematicians, such as Jean Dieudonné and Gustave Choquet, engaged in an in-depth discussion with American reformers, of whom Edward Begle was the most prominent, about future avenues for school mathematics; (2) it was the first in a series of similar meetings at which the reform was discussed and hence a booster of communication and internationalization in the

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This is a second paper based on a series of guest lectures at the University of the National Education Commission, Krakow, November 19–21, 2024. The first paper, titled *The rise and breakthrough of the international modern mathematics/New Math movement in the 1950s*, was published in the previous issue of this journal.

mathematics education community (Furinghetti and Menghini, 2023); and (3) it marked an acceleration of the movement in some European countries such as Belgium and France, and the starting point in other European countries; the other continents would follow shortly after. In the words of Bjarnadóttir (2008): "The Royaumont Seminar can be seen as the beginning of a common reform movement to modernize school mathematics in the world" (p. 145).

Following up on one of recommendations of Royaumont, several countries set up classroom experiments with modern mathematics in the early and mid-1960s. They often received considerable attention and appreciation at international forums (see, e.g., OECD, 1964). Particularly noteworthy are the audacious approaches of Max Beberman, Zoltán Dienes, and the Belgian Georges Papy, combining mathematical rigor with innovative pedagogies. At that time, the introduction of modern mathematics in schools was already in full preparation in most Western European countries, especially those that were members of the "Organisation for Economic Co-operation and Development" (OECD)[1], the organizer of the Royaumont Seminar.

During the second half of the 1960s, modern mathematics spread rapidly worldwide. The reform debate had already reached a number of countries in Eastern Europe, Latin America, Africa, and Asia in the early 1960s, countries that were not part of the original OECD. In 1978, the International Commission on Mathematical Instruction (ICMI) published a report "Change in Mathematics Education since the late 1950s", which included 16 countries around the globe, with the exception of countries in South America (Freudenthal, 1978). The report documented the reform efforts in each participating country in the preceding two decades, the different directions, the varying degrees of success, and the influences of educational systems.

The above and more recent publications (e.g., De Bock, 2023) show that modern mathematics/New Math was not a homogeneous set of reform proposals, even within continental Europe (Servais, 1975) and the US (Phillips, 2015). It was a flag that covered many loads!

In this article, we discuss the dissemination of modern mathematics/New Math after Royaumont, focusing on countries or regions where implementation was special in some way. We then turn to the critique of modern mathematics. Until the mid-1960s, critical voices received little hearing, but that quickly changed in the late 1960s. In some countries, including Belgium, it led to a fierce Math War that was fought mainly in public media. As a case study, we will take a closer look at the Belgian Math War. In the early 1970s, criticism also swelled at international forums which eventually led to the fall of modern mathematics.

2. Various implementations and faces of modern mathematics around the world

Within Europe, the cradle of the movement was in France and Belgium, two original OECD countries. In the wake of Royaumont, projects were soon set up

[1] The new name of the "Organization for European Economic Cooperation" (OEEC) after the US and Canada joined in late 1960.

in other OECD countries within Europe. The first aim was to reform secondary school mathematics for students preparing for university studies in mathematics, physics, or engineering. Modern mathematics reformers, however, believed that the same (improved) mathematics was useful for all students, and so in most countries the reform was implemented in all sections of secondary education, in some countries also in primary education (and even in kindergartens). The original OECD countries were followed by non-OECD countries in and outside Europe. In Latin America, Africa, and Asia, modern mathematics was often an import of Western models, although some countries developed their own interpretations of the reform.

2.1. Some notable examples in a nutshell

In the post-Royaumont era, Georges Papy, a professor of algebra at the Free University of Brussels, became the figurehead of the Belgian reform movement. He turned out to be an uncompromising leader, active in various domains: conducting experiments, writing textbooks (the famous *Mathématique Moderne* series), and re-training (*recycling*) mathematics teachers (De Bock and Vanpaemel, 2019). To coordinate all these actions, Papy founded the Belgian Center for Mathematics Pedagogy in 1961, probably the first-ever development center for mathematics education. Inspired by Bourbaki, Papy reshaped the content of secondary school mathematics by basing it upon the unifying themes of sets, relations, and algebraic structures. Meanwhile, he proposed an innovative pedagogy using multi-coloured arrow graphs, playful drawings, and "visual proofs" by means of drawings of film strips. In his textbooks, Papy also proposed a "solution" for a modernized teaching of geometry, based on axioms, logical deduction, and linear algebra. From the first year of secondary education, the synthetic axioms of affine (and later Euclidean) geometry were gradually introduced, culminating in the third year in the conceptualization of the plane as a two-dimensional Euclidean vector space. From the fourth year, this result was accepted as a "unique axiom" to further develop plane geometry and for building up space geometry; the "two" only had to be replaced with "three". With support from some powerful political friends, though not without controversy, he succeeded in pushing his vision almost integrally into the official programs of modern mathematics that were officially introduced in secondary education from September 1968. About ten years later, primary education would follow.

Papy inspired developments in a number of other countries, the best examples probably being France and Poland. In France, the Association of Mathematics Teachers of Public Education played a major role in implementing the reform even before Royaumont. In its *Bulletin*, Gilbert Walusinski, a former president and leading figure of the Association, had praised Papy's *Mathématique Moderne* and considered his actions as encouraging and inspiring examples for future developments in France (Walusinski, 1963). From 1964, the Association organized local courses and national television series to prepare teachers to the advent of modern mathematics in their classrooms (Barbin, 2012). In 1966, the French government created a Commission, chaired by the French mathematician André Lichnerowicz, to "rethink the teaching of mathematics" and to develop new programs.

The "Lichnerowicz Commission" urged teaching a "contemporary mathematics" through "active pedagogical methods". The official new programs appeared between 1968 and 1971 and were introduced progressively, starting from 1969 in the first year of secondary education. Roughly speaking, the French modern mathematics programs closely resembled those of Belgium: sets and relations linked to the teaching of logic, algebraic structures, the use of a modern mathematical language, the "construction" of the fields of rational and real numbers, a deductive-axiomatic approach to (first affine) geometry, eventually arriving at a linear algebra approach to two- and three-dimensional space.

As part of the aid after World War II, Choquet stayed for some time in Krakow (Błaszczyk et al., 2019). He made important contacts with different members of the Polish mathematical community but, according to a reviewer of this article, his conceptions would not have been appreciated at the Jagiellonian University. The main pioneer in the reform of school mathematics in Poland (and beyond) was Anna Zofia Krygowska. She was influenced by Piaget's ideas about an alignment between the fundamental structures of mathematics and the structures of cognitive development (Semadeni, 2023). In 1957, Krygowska joined the International Commission for the Study and Improvement of Mathematics Teaching and organized a meeting of this group in Krakow in August 1960. This meeting brought her in contact with Papy, whose ideas on *mathématique de base* ["base mathematics"], referring to the fundamental role of sets and relations in modern mathematics, influenced Krygowska's thinking for a long time. Krygowska participated in a ministerial commission for the modernization of Polish curricula to be implemented in 1963–1971, first in grades 5–8 and then in grades 1–4 of secondary school. While mathematics in grades 5–8 remained quite traditional, the curriculum for grades 1–4, implemented from 1967, was strongly influenced by *Mathématique Moderne*, especially in the approach to geometry, for which an axiomatic-deductive approach was adopted. In an accompanying textbook, Krygowska developed the conception of global deduction in geometry based on a fixed axiom system. However, she did not use the term "axiom", but called it a "basic property". Krygowska's approach also assumed two other systems and associated symbolisms: the naive theory of sets and the real number system. Krygowska's geometry textbook was widely criticized because students experienced severe difficulties in learning from it. In the late 1960s, Krygowska developed a modern program for primary grades 1–4, but her proposals were only partially adopted.

The Royaumont Seminar also initiated mathematics curriculum change in the United Kingdom, taking influences of continental Europe and the United States (Rogers, 2023). Several reform projects were established in the early 1960s, but the School Mathematics Project (SMP), launched in 1963, would become the most successful project for secondary schools. As part of these projects, textbooks were written by teams of practicing teachers and in-service training courses were organized. However, unlike similar projects in the US, there was no substantial backing from university mathematicians, nor was any public (state) money invested in these projects. In terms of content, the British projects did not fall into the excesses of over-abstraction as in some continental or American versions of the reform. Rather than being set-driven and axiom-driven, they were essentially practical in nature, with examples from the "real world" (Geoffrey Howson in Karp, 2008). While there was some "modern" pure mathematics covered, such as groups, vectors, and transformations, there was an additional focus on "modern" applied mathematics (data collection and representation, statistics, linear programming, binary systems and codes, flow diagrams, computer programming, ...), topics that were also advocated at Royaumont, but which were often not addressed in continental reform programs. The SMP textbooks in particular were quite pragmatic and, in a modified form, would survive the New Math era. For primary education, the Nuffield Mathematics Project, launched in 1964, became the most important reform project. The aims of this project included:

> [T]o introduce [children from 5 to 13] gradually to the processes of abstract thinking, and to foster in them a critical, logical, but also creative, turn of mind.
>
> A synthesis will be made of what is worth preserving in the traditional work with various new ideas. (Moon, 1986, p. 134)

Hungary represents a particular case in the European reform movement. The leader of the reform in Hungary, Tamás Varga, elaborated a project named "complex mathematics education", which promoted a "guided discovery approach" to mathematics education (Gosztonyi, 2023). In contrast to most other countries, it is an example of a bottom-up reform project: From 1963, the (experimental) reform efforts started at the primary level; the extension to secondary education followed from 1971. The project was influenced by contemporary international modern mathematics/New Math ideas, but at the same time strongly rooted in the Hungarian mathematics education culture and tradition (focussing on problem solving and heuristic methods, stimulating students' creativity and inquiry, the use of manipulative tools and games, ...). Varga's experimental curriculum consisted of five main domains: sets and logic; arithmetic, algebra; relations, functions, sequences; geometry, measures; combinatorics, probability, statistics. These topics were already introduced from primary school and returned year after year through a so-called spiral approach: ever deeper, richer, and more precise (Servais, 1975). In particular, Varga's pioneering work on teaching logic, combinatorics, and probability (Varga, 1972) was internationally recognized (Gosztonyi, 2020). In the early 1970s, Varga's project was chosen by the Hungarian government as the basis for a new prospective curriculum, which implied dissemination on a much larger scale, but at the same time provoked significant resistance. The Soviet Union did not escape the reform movement of the 1960s either. The Soviet movement was led by the eminent mathematician Andrey Kolmogorov and is usually referred to as the Kolmogorov reform. According to Karp (2014), the reform proposals resembled what was happening in other countries: basing mathematics education on set theory, introducing vector and transformation geometry and combinatorics, paying more attention to functional-theoretical questions, and less attention to technical skills. Borovik (2023) argued that, in part, the reform followed a different strategy than elsewhere in the world: it was essentially an (unsuccessful) attempt to transfer content and methods that proved successful for training gifted and engaged students in specialized mathematics boarding schools to mainstream education.

In the early 1960s, modern mathematics/New Math also reached Latin America, Africa, and Asia. In Latin America, the reform was mainly driven by US mathematicians, Marshall Stone in particular, although there were also European influences. At that time, the US invested heavily in what it considered its "backyard" for political reasons: the fear that Latin America would "go communist" (de Carvallo, 2014, p. 353). The movement in Latin America was nurtured by the Inter-American Committee of Mathematics Education (IACME) and the conferences that this body organized. At IACME1 (Bogotá, Colombia, 1961), with the participation of leading US mathematicians (Stone, Begle, Howard Fehr, ...) and French "Bourbakists" (Choquet, Laurent Schwartz, ...), the modernization was defined (changing the Euclidean perspective in geometry education in favor of a linear algebra approach, teaching mathematics through the study of unifying concepts and basic structures, ...) and the way forward was paved. At the follow-up meeting, IACME2 (Lima, Perú, 1965), the ideas were reinvigorated and reform tactics, including teacher training and textbook production or translation, were reviewed. Already at IACME3 (Bahía Blanca, Argentina, 1972), the interest in New Math had waned, which would give room for new conceptions of mathematics education in Latin America (e.g., ethnomathematics). Looking back at the movement in Brazil, D'Ambrosio (1991) argued that the view on New Math was not coherent, but "a concoction of ideas from around the world [... which] generated a curriculum based on inconsistencies of various kind" (p. 71).

In Africa, many local reform movements took inspiration from the US or the UK. Best known is the African Mathematics Program (AMP), better known as the Entebbe Project. Led by William Martin, chair of the mathematics department at MIT, the AMP organized a workshop in 1962 in Entebbe, Uganda, with 54 participants from 13 countries, including 24 educators from 11 English-speaking African countries. The ensuing project, funded by the Ford Foundation and the US Agency for International Development, produced more than 60 volumes that were piloted in Ethiopia, Ghana, Kenya, Liberia, Malawi, Nigeria, Sierra Leone, Tanzania, Uganda, and Zambia. According to Swetz (1975), the entire project "was American dominated with the writing strongly influenced by advocates of the School Mathematics Study Group (SMSG). [...] The result was a blackfaced version of SMSG mathematics" (p. 6). But, he continues, "to challenge the American 'menace', two competing British-oriented writing groups were formed. The Joint Mathematics Project was begun in West Africa and the East African School Mathematics Project in East Africa; both projects emulated the British counterpart of SMSG, i.e., the School Mathematics Project (SMP)" (pp. 6–7). In the North African country of Morocco, elements of modern mathematics were introduced into the official programs of upper secondary education as early as 1962. According to some Moroccan authors, this did not respond to a national need: France, the former colonial power, considered Morocco as a space to experiment with modern mathematics before it was incorporated into the French curricula (Laabid, 2023).

The modern mathematics/New Math movement in Asia is barely documented in the international scholarly literature. Gosztonyi et al. (2023) mention that Japanese mathematics education in the 1960s was influenced by New Math, resulting in the incorporation of set theory, algebraic structures, and linear transformations into secondary school lessons. It is likely that New Math in Japan was inspired by Western models (for example, the first volume of Papy's *Mathématique Moderne* was translated into Japanese), but detailed English-language accounts of this reform are lacking. Begle (1968) states that SMSG materials were translated and/or adapted in many countries, including Turkey, Taiwan Australia, and India, but does not provide information on how or to what extent these materials were used in these countries. Recently, Siu and Wong (2023) shed light on the reform movement in Hong Kong. Initially following in the footsteps of the British (SMP), Hong Kong gradually developed its own interpretation of reform.

2.2. An attempt to characterize the reform

As exemplified in the previous section, modern mathematics/New Math had many different faces. The question arises whether a unique and comprehensive characterization of *the* modern mathematics movement is possible at all. Was there a common core to which all national (subnational/regional) reform efforts adhered? At the risk of being one-sided and partly betraying a well-intentioned reform movement, we make an attempt anyway.

First, there was the naive theory of sets, a "new" framework for a unified presentation of mathematics and a starting point for teaching, both in content and in method. In the words of Papy (1961):

> While the space of Euclid could for a long time serve as the framework for a unified presentation of basic mathematics, it can no longer today, but its role can now be fulfilled by the *universe of sets*. Moreover, as it has been proved by experiments carried out in America, England, Russia, Poland and in our country, the teaching of the *basic notions of set theory* fascinates young students. It therefore seems inevitable to propose that topic as *starting point* in secondary education. (p. 21)

The wider public associated the new mathematics primarily with the language of sets and the iconic representations of Venn and arrow diagrams that began to dominate textbooks from the mid-1960s.

Second, more essential than "sets and arrows", was the determination to shape mathematics education from the standpoint of mathematical structure(s), starting with poor structures and gradually building up more rich structures. The latter led to a global algebraization of mathematics education, to the prioritization of the affine viewpoint in geometry, and to the deletion of most of the synthetic (Euclidean) geometry of figures. The concept of structure was central to Bourbaki's attempts, beginning in the 1930s, to reorganize mathematical science, but after World War II, Bourbaki's structure-based view of mathematics was seen as a tool to organize and promote school mathematics. Several reformers of the 1960s referred to Bourbaki as their main inspiration, e.g.:

> Euclid's Elements exposed the basic mathematics of his time, about 300 BC. The monumental work of Nicolas Bourbaki presents, at the

> highest level, the basic mathematics of today. The "MMs[2]" want to expose the Elements of today's basic mathematics for adolescents ... and people of any age and schooling who wish to initiate themselves in the mathematics of our time. (Papy, 1967, p. vii)

Because the structure of the science of mathematics was used as a guiding principle to develop mathematics education, Hans Freudenthal (1991) labelled this approach as a structuralist view on mathematics education.

> In the nineteen sixties and seventies of our century, under the name of New Math, the structuralist view was advertised and propagated. [...] On behalf of the pre-structured mathematics to be taught, a correspondingly structured world was invented of Venn diagrams, arrow schemes, "games" and so on, to be mathematised by the learner. This was, indeed, a kind of horizontally mathematising activity, yet it started from an *ad hoc* created world, which had nothing in common with the learner's living world. It was mathematics taught in the ivory tower of the rational individual, far from world and society. (p. 135)

The emphasis on mathematical structure (rather than the mastery of specific knowledge or technical-computational skills) implied the introduction of new content, materials, and practices in school mathematics. Although specific features varied across countries and regions, and obviously with level of education, we attempt to list some main features of mathematics curricula in the 1960s and 1970s.

- So-called "fundamental" concepts like sets and relations became the starting points; "richer" concepts were described in terms of these concepts (e.g., geometrical objects were defined as "sets of points");
- Concept and problem representation were directed to the use of Venn diagrams and arrow graphs; students were discouraged from making their own problem visualizations;
- Recommended curricula were oriented from the abstract to the concrete (e.g., in geometry: first points, then lines, and finally "rich" geometrical figures);
- Precise formulations, exact definitions, and correct symbol use received much attention in verbal and written explanation and communication (e.g., a clear distinction was made between "numbers" and "numerals");
- There was a focus on conceptual mathematical understanding (e.g., by studying number systems with bases other than 10) and basic laws (commutativity, associativity, ...), rather than on computational fluency and number facts;
- Linear algebra became a "royal road" (Choquet, 1964, p. 11) to affine and, thereafter, Euclidean geometry; much attention was given to geometric transformations and their underlying structures, and less to geometric problem solving.

[2] Papy's Mathématique Moderne series.

Reformers of the 1960s repeatedly hinted at the "usefulness" of the new mathematics. For example, in the *Charte de Chambéry*, an important French reform document prepared by the "Lichnerowicz Commission", we read:

> Contemporary mathematics is useful in many domains: theoretical physics of course, but also computer science, operational research, stock management of companies, organization charts of big administrations, planning for major projects, sociology, linguistics, medicine (diagnosing), pharmacy... (Charte de Chambéry, 1968)

However, the claimed usefulness of modern mathematics was not reflected in the textbooks of the period. The reason is evident: for the structures of modern mathematics, such as groups, fields, or vector spaces, no *applications* were available to the young-age students to whom these structures were taught. What textbooks of that era did offer were *illustrations* of the new concepts: simple, artificially designed situations, often presented as games, which were easy to understand and which served as a starting point for abstraction and theory building. Papy (1968) called this the *mathematization of situations*, which he considered more important than addressing "real-life" applications:

> Students are immediately accustomed to an approach which is essential for applications: The mathematization of situations. Obviously, it is difficult to predict the kind of mathematics that will be used by the students later. In the modern world, mutations are common. Many people, during their lifetime, have to change profession several times and, in any case, technical skills in their own profession. Mathematics does not escape from this phenomenon. [...] We do not know how to predict which situations will be mathematized later, nor which mathematics will be used for that purpose, but we know that the mathematization of situations will remain fundamental. It is therefore essential to accustom our students, from the beginning, to this important strategy of the mind. (pp. 7–8)

3. A failed reform?

The first criticism of modern mathematics/New Math appeared in the early 1960s. Critics at that time were lone wolves sailing against the tide but unable to turn the tide. By the late 1960s, the criticism in some countries grew into organized acts of opposition, even into a real Math War. International systematic criticism by leading mathematicians such as René Thom and Morris Kline gave the movement its final death blow in the first half of the 1970s.

3.1. Critiques from the early 1960s onward

Not surprisingly, the first criticism came from those responsible for applying mathematics: professors of mathematics in engineering faculties. An interesting example is Belgian Léon Derwidué. Derwidué pointed to the lack of help that Bourbaki-style mathematics could provide to users of mathematics:

> For almost ten years now I have lived in engineering circles, I consulted their journals, I tried to help them solving their mathematical problems, but never I encountered the use of Bourbaki-style mathematics, except in forms so obvious that they are not worth mentioning. (Derwidué, 1962a, p. 6)

And even for students who would later devote themselves to pure mathematics, Derwidué was not convinced that learning mathematics based on a Bourbaki-like presentation was a good starting point:

> Moreover, is it not appropriate to begin the mathematical education of every teenager, even those destined for the purest mathematics, with the useful and concrete aspect, which in its further development will naturally reveal problems for which rigorous treatment seems necessary? (Derwidué, 1962b, p. 10)

Moreover, he argued, if we want to carefully prove all the details before we can move on, we will get stuck in the foundations and, due to lack of time, students will finish their secondary school education with a mathematical package that is largely inadequate for their further studies or for their professional life (Derwidué, 1962b).

A more famous representative of the opposition front is mathematician Morris Kline, an outspoken and persistent critic in the US. As early as 1955, he had stated that the stress on "rigor" in Begle's calculus textbook had reached "the heights of absurdity" (Kline, 1955, as cited in Roberts, 2023, p. 30). In a virulent attack under the headline "Math Teaching Reforms Assailed as Peril to U.S. Scientific Progress", Kline (1961) argued that the reform efforts as led by the SMSG were "wholly misguided", "sheer nonsense", and attempts to replace the "fruitful and rich essence of mathematics with sterile, peripheral, pedantic details" (DeMott, 1964, as cited in Kilpatrick, 1997, p. 956). More coordinated action was taken in 1962. Kline, along with 64 other noted American and Canadian mathematicians (including André Weil, the intellectual leader of Bourbaki), adopted a well-considered memorandum "On the Mathematics Curriculum of the High School" (Ahlfors et al., 1962). It echoed many of the criticisms and proposals that Kline had voiced in earlier publications, in particular that the curriculum should meet the needs of all students and not just those of bright, mathematics-bound students, that abstract mathematics should not be introduced to students too early, that they should see the connections between mathematics and the other sciences, that intuitions and conjectures should come before formal proof, that mathematical ideas should be introduced wherever possible as they had arisen genetically, and that traditional school mathematics should not be completely replaced by so-called modern mathematics.

From the mid-1960s, criticism of modern mathematics/New Math was no longer confined to academic circles. However, the public discourse, as voiced in the press or through other public media, remains an under-researched aspect of the reform movement (Zelbo, 2025). That the school mathematics reforms were alive among the wider public is shown, for example, by the fame of the "New Math" song of 1965 by satirist Tom Lehrer in the US. The song is basically a mathematics lesson for parents who could no longer assist their children with their homework. The topic of the song is an algorithm for subtraction in the number system with base eight, with Lehrer observing that base eight is "just like base ten really – if you're missing two fingers". Some calculation errors are made, but actually that is not a big problem because: "in the new approach, as you know, the important thing is to understand what you're doing, rather than to get the right answer (Lehrer, 1965).

In Europe, where modern mathematics was introduced into schools a little later than in the US, the criticism in the public media came mostly in the late 1960s and early 1970s. Moon (1986) wrote that "Modern maths, in France, as elsewhere makes a good media story" (p. 98), and exemplified his assertion with a reference to *Le cauchemar des maths modernes* [The nightmare of modern mathematics], a powerful polemical article in *L'Express* of February 6, 1972, by Roger Apéry, a professor of mathematics at the University of Caen, which provoked a reaction from Lichnerowicz. Another well-known example is the article *Macht Mengenlehre krank?* [Does set theory makes sick?] in the leading German weekly *Der Spiegel* of March 25, 1974. It prompted Freudenthal's reaction that it has since been forbidden to teach set theory in schools in most German *Länder* [states] (Stichting-Lodewijk de Raet, 1983). But as noted above, there is hardly any systematic scholarly research on New Math criticism in the public media. An interesting case that has been studied is Belgium (De Bock, 2023), which we summarize in the next section.

3.2. A case study: The Belgian Math War

Protagonists in the Belgian Math War were, along one side, Georges Papy, the undisputed leader of the Belgian modern mathematics movement, backed by "his" Belgian Center for Mathematics Pedagogy. These so-called "Papy-ists" opposed the "anti-Papy-ists", represented by the aforementioned Léon Derwidué, professor of mathematics at the polytechnic school in Mons, and Charles-François Becquet. Becquet was a teacher and later an inspector of technical secondary education. Technnical schools in particular complained about the loss of geometric representations, the emphasis on logic and abstract concepts, and the isolation of the new curriculum from other subjects, such as technical drawing, which required an understanding of spatial forms.

The origin of the Belgian Math War was a decision in 1965 by Henri Janne, the then Belgian Minister of Education, to make modern mathematics compulsory in all first years of Belgian secondary education from September 1, 1968. Janne was a French-speaking socialist, former rector of the Free University of Brussels and a political friend of Papy. In March 1966, however, the political climate in Belgium changed drastically: A new government without socialists was formed and the Flemish liberal Frans Grootjans became Minister of Education. Grootjans had soon received a delegation of professors from the Science and Engineering faculties who had asked him to withdraw Janne's earlier decision. It brought about some panic in the Papy camp (and hope among his opponents). In a letter to Papy, dated January 4, 1967, Grootjans stated: "You can be assured that I will not make any decision without being informed by all the authorities responsible for the teaching *and application* of mathematics" (our italics).

To obtain the desired advice, Grootjans established two national study commissions: a University Commission and a Commission for Secondary Education. The University Commission, installed on March 20, 1967, had to advise the Minister on the necessity to radically change the secondary school mathematics curriculum of the scientific streams to meet the needs of the University, higher education, and trade and industry. In addition, this University Commission had to broadly define the crucial mathematical knowledge that secondary education had to provide to meet the identified needs. The Commission for Secondary Education, consisting of inspectors, informed teachers, and experts from the Educational Administration, had to develop a final mathematics program based on the advice of the University Commission. Debates in the University Commission were most tumultuous; for example, the delegation of the University of Ghent, opponents of "Papy's method", left the last meeting before the final vote, which consequently fell in favor of the "Papy-ists"... The University Commission's advice, strongly in favor of the reform, was reached on September 12, 1967. Soon thereafter, the Commission for Secondary Education agreed on a new program for the first year of secondary school that strongly reflected Papy's views.

On April 11, 1968, in accordance with the advice given, Grootjans confirmed Janne's earlier decision: Modern mathematics would become mandatory from September 1, 1968. However, this ministerial decision only concerned *general* secondary education; therefore, nothing had yet been decided about modern mathematics in technical secondary education. Moreover, on June 17, 1968, a new government was formed with two new Ministers of Education: The Fleming Piet Vermeylen and the Francophone Abel Dubois, both socialists. The newly appointed Ministers soon allowed some schools of general secondary education "not yet ready to implement the new curricula" to postpone the introduction of modern mathematics by one year (Noël, 1993). Could the decision of Grootjans be reversed? A time of uncertainty followed for "Papy-ists", a time of hope for his opponents. This uncertainty led to a fierce Math War in 1968–1969 in which both "Papy-ists" and "anti-Papy-its" mobilized all those involved in the reform, including parents, for large-scale information meetings, hearings, and colloquia in major Belgian cities.

These "public debates", actually open quarrels between proponents and opponents of the reform, were not uplifting for mathematics education, but did enjoy considerable press coverage. Walter Schwilden, a journalist for *Le Soir*, one of Belgium's most widely read newspapers, who reported on a large-scale meeting in Brussels, did not hide his outrage:

> Personally, as an observer not immediately concerned by the problem, I left the Albert I conference hall [on April 27, 1969] disgusted by the painful spectacle that was offered to us. [...] In Brussels, on Sunday morning, it was only a new stage in the confrontation, the painful spectacle of a foolish battle that burns up energies, that ruins the collaboration between the different levels and types of education, that suffocates the trust demanded from parents by teachers and their

> masters, that calls for a nasty smile, a disillusioned comment. Who should open the abscess, empty the quarrel?

Especially in the Francophone press, the article headlines were telling: “La guerre des maths aura-t-elle lieu?” [Will the maths war take place?] (*Spécial*, March 6, 1968), “Sur le front des maths” [On the maths front] (*Pourquoi Pas?*, August 29, 1968), “La guerre des math: Papystes contre antipapystes” [The math war: Papy-ists versus anti-Papy-ists] (*Le Soir*, December 17, 1968), “Des cobayes pour les Papystes” [Laboratory animals for Papy-ists] (*Spécial*, April 9, 1969), “Ŕ quand un cessez-le-feu et une commission d’armistice? Le pénible spectacle offert par la ‘guerre des math’ ” [When will there be a ceasefire and an armistice commission? The painful spectacle offered by the “math war”] (*Le Soir*, April 27–28, 1969). The leftist press was mostly sympathetic to the reform initiated by Papy and his Center, not only because Papy was a socialist, but also because there was a vague belief in the emancipatory power of the project. After all, with modern mathematics, every student, without parental help, had to start from scratch, regardless of his or her social background.

The Belgian Math War was eventually won by Papy and his proponents. Minister Dubois stuck to the decision of his predecessors. On April 27, 1969, he declared:

> The new curriculum is the Belgian version, very pragmatic and very adaptable, of a mathematical conception which is now being introduced in all industrialized countries; [...] it constitutes a clear obligation for all schools run by the state; no one, my predecessors nor myself, has ever envisaged reconsidering it.

When from September 1, 1969, all students of the first years of secondary education, both in the general and in the technical sections, without any exception, were subjected to modern mathematics, the late 1960s Math War ended quickly. The official programs of the technical sections were similar to those of the general sections except for a few details. For the socialist Dubois, this was a deliberate decision, even a matter of principle, namely an occasion to eliminate, maximally, divisions between different types of education (and thus an opportunity to upgrade the status of the technical schools). In his own words:

> It would be unacceptable if not [the same programs were applicable] and that, with the introduction of the new program, the harmful division between general and technical education, each in their respective fortresses, would be continued.

But who laughs last, laughs best? Already in the early 1970s, Papy realized that the reform movement was stagnating; in 1972, he declared to the press “le vaisseau de la réforme s’est embourbé” [the ship of the reform is stranded]. Papy, whose influence in Belgium and abroad had waned from the 1970s, no longer publicly defended his brainchild. From the early 1980s, modern mathematics in Belgium began to shake on its foundations and its final fall appeared inevitable (De Bock and Vanpaemel, 2019).

3.3. The ending at the international forum in the early 1970s

The critical analysis of the international modern mathematics/New Math movement in the early 1970s was led primarily by René Thom, a prominent French mathematicician who was awarded a Fields Medal in 1958 for his work in topology, but is best known as the founder of catastrophe theory. Based on his mathematical background, Thom devoted himself to philosophy in the last decades of his career and also became thoroughly involved in debates about mathematics education, particularly about the implementation of modern mathematics curricula in France and elsewhere (Papadopoulos, 2022). A seminal article, entitled "Modern mathematics: an educational and philosophical error?" (Thom, 1971)[3], was published in *Scientific American* and prompted a response from Dieudonné (1973). In defiance of the trend at the time to focus mathematics education primarily on set theory and algebraic structures, Thom showed himself a defender of classical Euclidean geometry: "Geometry is a natural and possibly irreplaceable intermediary between ordinary language and mathematical formalism". And he added: "There is hardly any doubt that, from a psychological and, for the writer, ontological point of view, the geometric continuum is the primordial entity" (p. 698). Referring to critics who objected to the axiomatics of Euclidean geometry for being flawed and not rigorous, Thom argued:

> There is no rigorous definition of rigor. [...] Rigor (or its contrary, imprecision) is essentially a *local* property of mathematical reasoning. [...] Axiomatization is the work of specialists and has no place in secondary or college teaching [...]. All this explains why the reproaches of inconsistency directed at Euclidean geometry are irrelevant; they do not touch the validity of local intuitive reasoning. [...] Euclidean geometry is the first example of the transcription of a two- or three-dimensional spatial procedure into the one-dimensional language of writing. In this, Euclidean geometry applies a rigid, precise situation, a procedure which is already present in everyday language. (pp. 697–698)

Dieudonné responded with a defense of modern mathematics, particularly the approach to geometry from linear algebra:

> The basic principle of modern mathematics is to achieve a complete *fusion* between "geometric" and "algebraic" ideas, opposing "geometry" to "algebra" as Thom does is simply meaningless [...]. Once the basic theorems of Euclidean geometry have been established by linear algebra (*without coordinates*, of course!), there is nothing to prevent the bright student from tackling the classical problems on triangles or conics if he is so minded. (p. 19)

In 1972, Thom was invited to deliver a plenary address at the Second International Congress on Mathematical Education held in Exeter, UK (Thom, 1973). He opened as follows:

[3]The article was the translation of an article in French published the year before. The English-language version in particular had major international impact.

> The future historian of mathematics will not fail to be amazed by the extent of the movement of the 1960s known as Modern Mathematics. [...] Only dogmatic spirits (and they are not lacking among "modernists") can believe that there is in these questions a truth capable of being logically established and before which one needs must bow. (p. 194)

At the heart of his talk, he returned to the issues of formalization, axiomatics, and rigor. The ideal of absolute rigor that "modernists" strive for does not exist and, moreover, this is not an essential problem for the teaching of mathematics.

> *The real problem which confronts mathematics teaching is not that of rigor, but the problem of the development of "meaning", of the "existence" of mathematical objects.* (p. 202, italics in original)

Among the New Math critics in the first half of the 1970s, we must again mention Morris Kline. In his book *Why Johnny Can't Add: The Failure of the New Math* (Kline, 1973), written in a simple and clear yet sharp style, the author collected his criticisms of New Math. It became a bestseller in its genre and was translated into several languages. Also Hans Freudenthal should not be left out in this shortlist of famous New Math opponents. Freudenthal converted quite early to the critical camp (Freudenthal, 1963). As a president of ICMI in 1967–1970, he played a major international role. Nationally, he could prevent modern mathematics from gaining a firm foothold in the Netherlands.

Two further comments can be made about the criticisms of modern mathematics/New Math that caused or accelerated its decline.

First, criticism sounded first and loudest in countries where the reform was most radically implemented. The US, France, and Belgium are prototypical examples. In countries where the reform was implemented more pragmatically/less dogmatically, certain aspects of the reform remained alive even to this day: In the UK, mathematics education projects based on the SMP materials continue to be developed (see, e.g., `https://smp2.co.uk/`), and in Hungary, Varga's method is still a vibrant source for designing contemporary mathematics education (Gosztonyi, 2020).

Second, in almost all countries, modern mathematics/New Math was implemented in a "top-down" order (Moon, 1986): Curriculum reform first entered universities, then was advocated for the scientific streams of upper secondary education. Next, the "non-scientific" streams of general education, technical education, and the middle school came into the reformers' spotlight. Finally, within a space of three to five years, arithmetic-mathematics instruction in primary school became involved in the reform. In several countries, criticisms of modern mathematics/New Math followed the reverse ("bottom-up") order: Excesses of the reform were first identified and taken seriously in primary education, and only later in secondary education. Belgium was a typical example of this (De Bock and Vanpaemel, 2019).

4. To conclude

In most countries, modern mathematics/New Math did not survive the 1970s and the retrospective assessment was typically (very) negative. In its issue of July 14, 1999, *Time Magazine* listed New Math as one of the 100 worst ideas of the century. In some countries, the introduction of New Math has indeed caused considerable damage to existing rich traditions in the teaching of mathematics to children, and some effects are still being felt today. Gruszczyk-Kolczyńska (2017) has presented a number of interesting observations in this context.

But the reform movement of the 1960s may also have had a number of positive effects. It generated national and international momentum in the community of mathematics teachers (and other people involved in mathematics education). There were lively debates in several countries about the upcoming reform of mathematics, a discipline previously considered unchangeable. Committed teachers felt connected as collaborators in an ambitious and valuable educational project that often crossed ideological boundaries. From the end of the 1960s, partly out of the ashes of the modern mathematics movement, the amount of international cooperation grew, and *mathematics education* became recognized as an autonomous scientific discipline. Anna Zofia Krygowska played an important role in this process. Under Freudenthal's presidency, ICMI adopted an agenda which fostered a new and refreshing dynamic in thinking about and researching mathematics education. The anchor points were the establishment of the International Congresses on Mathematical Education (the first took place in Lyon, France, in 1969) and the creation of the journal *Educational Studies in Mathematics* (launched in 1968). Modern mathematics was by no means an overall success, but it was at the root of developments that probably would not have occurred without it.

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