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Dirk De Bock

The Rise and Breakthrough of the International Modern Mathematics/New Math Movement in the 1950s*

Abstract. The origins of modern mathematics in continental Europe and New Math in the United States can be situated in the early 1950s, and although both educational reform movements share common sources and motives (and are often seen as one and the same movement), they initially developed independently of each other. In this article, we examine the early roots of these movements: the context, actors, and motives. In Europe, the debates were mainly related to scientific innovations, such as Bourbaki's work in pure mathematics and that of Piaget in developmental psychology. The US movement was more strongly rooted in socio-political and economic motives, such as the demand for mathematically skilled labor, and was government-driven early on. In 1959, key European and American reformers met for the first time at a seminar in Royaumont. This seminal event marked the beginning of the worldwide spread of modern mathematics/New Math.

1. Introduction

Modern mathematics, also known as New Math, which bases the teaching of mathematics on set theory and mathematical structures, was a radical educational reform movement that originated in the 1950s, reached its height in the 1960s, and declined in the early 1970s. In recent years, scientific interest in the phenomenon of modern mathematics/New Math has increased significantly. However, although

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This is a first paper based on a series of guest lectures at the University of the National Education Commission, Krakow, November 19–21, 2024. The second paper, titled "Modern mathematics/New Math: From its heights in the 1960s to its fall in the early 1970s," will follow in the next issue of this journal.

the modern mathematics/New Math movement had a clear international character, most historians focus on developments in their own country or region. Such studies demonstrate the national embedding of the movement, or as Kilpatrick (2012) wrote, "The more school mathematics is internationalized, the more clearly its national character is revealed" (p. 570). But evidently, national histories are interconnected, and research aimed at revealing differences and similarities between country-specific motifs and manifestations is most welcome.

The British educationalist Bob Moon (1986) was a pioneer in that respect. In his seminal book, *The 'New Maths' Curriculum Controversy: An International Story*, Moon presented five case studies of the reform (The Netherlands, France, England and Wales, West Germany, and Denmark) against the background of the progressive socioeconomic climate of the 1960s. More recently, the author of this article has edited a volume from a similar perspective, with analyses of the reform in 20 countries or regions around the globe, focusing on the preparation and implementation of the reform in the US and Western Europe, and on the dynamics of its subsequent spread to all continents (De Bock, 2023). However, despite the two aforementioned books and a few individual papers, comparative research on modern mathematics/New Math remains rare. We endorse Oller-Marcén's (2022) assertion that "a detailed cross-national comparison of the process of its introduction, implementation and eventual disappearance is still to be made" (p. 191).

In this article, we outline the origins of the modern mathematics movement in continental Europe, with special attention to the leading countries France and Belgium, and the New Math movement in the US, and point out some parallels and differences. We thus focus on the evolution of both movements in the early to mid-1950s, culminating in the legendary Royaumont Seminar held in late 1959. In Royaumont, European mathematicians from the milieu of Bourbaki, Jean Dieudonné, and Gustave Choquet being the most prominent, were brought together with American reformers such as Edward Begle, director of the governmentally strongly supported School Mathematics Study Group. However, the oft-repeated claim that the New Math originated in the US and crossed the Atlantic in 1959 (see, e.g., Nadimi Amiri, 2017) is unjustified: As we will outline, already in the early 1950s, ideas were launched in Europe to reorganize the teaching of mathematics according to the model that Bourbaki had developed for the science of mathematics from the end of the 1930s onwards. A psychological justification for this reorganization was provided by Jean Piaget's work in developmental psychology.

2. The origins of the European modern mathematics movement

The organization that initiated the European reform movement was the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM) [International Commission for the Study and Improvement of Mathematics Teaching], formally established in 1952 after Caleb Gattegno, an Egyptian-born mathematician and psychologist, had paved the way for it in the previous two years. We reconstruct the debates from the early meetings of this group, particularly the meeting in 1952 where the Bourbakists met the Swiss psychologist Piaget. Debates resulted in the assumption of an alignment of mathematical and mental structures, which became a main argument for the reform of mathematics education in Europe.

2.1. The debates within CIEAEM in the early 1950s

In April 1950, Gattegno brought together an international group of experts in mathematics, psychology, and education, including some experienced mathematics teachers, in Debden (UK). Although the number of participants was limited, the range of competences had to allow for "a thorough reconsideration of the whole problem of the child and mathematics" (Gattegno, 1947, p. 220). This meeting, followed by two similar meetings in 1951, one in Keerbergen (Belgium) and one in Herzberg (Switzerland), led to the official founding of the CIEAEM in La Rochette sur Melun (France) in April 1952.

In La Rochette, the foundations were laid for the "modern mathematics" movement in Europe. The meeting was not an accidental encounter between top mathematicians and top psychologists/epistemologists of that time; on the contrary, the meeting was carefully prepared by Gattegno who, as a holder of doctorates in mathematics and in psychology, was familiar with the recent developments in these fields, in this case particularly the work of Bourbaki and Piaget. Already at the Debden meeting, Gattegno would have announced: "I will have the Bourbakists, I will have Piaget, I will have Gonseth" (Félix, 1986, p. 26).

The debate in La Rochette was initiated by Dieudonné who outlined Bourbaki's points of view, paying particular attention to the origin and essence of structures in modern mathematical science (Félix, 1986). He argued that structures are by no means artificial constructs that appear out of nowhere; they are "explicitations" of ideas that were already present in the work of great mathematicians of the past, implicitly and under different guises, but which were not yet recognized as such. Their role in mathematical research was clarified by André Lichnerowicz: "A structure is a tool that we search for in the arsenal we have at our disposal. It is not at this stage that it is created" (cited in Félix, 2005, p. 82). Choquet and Lichnerowicz also testified about how they actually used structures in their research (Félix, 1985, 1986, 2005). For the teachers in La Rochette, the vision of the Bourbakists and the way they practiced mathematics was nothing less than a revelation. After World War II, Bourbaki's work was known to research mathematicians, but most secondary school teachers, even those who had graduated in mathematics, were completely ignorant of this "modern" evolution within mathematics.

During the discussion, Dieudonné emphasized that the Bourbakists were not dealing with questions of a philosophical or metaphysical level; only common logic was used (Félix, 1986). Associations of mathematical structures with extra-mathematical constructs were not suggested by Bourbaki, but they were established by Piaget who explicitly related Bourbaki's structures to the mental operations through which a child interacts with the world (Piaget, 1955).[1] More

[1] Piaget (1955) was the summary of his presentation in La Rochette (1952), as mentioned in

specifically, Piaget identified the fundamental structures and stages of early mathematical thinking with the mother structures in the work of Bourbaki:

> Now, it is of the highest interest to ascertain that, if we retrace to its roots the psychological development of the arithmetic and geometric operations of the child, and in particular the logical operations which constitute its necessary preconditions, we find, at every stage, a fundamental tendency to organize wholes or systems, outside of which the elements have no meaning or even existence, and then a partitioning of these general systems according to three kinds of properties which precisely correspond to those of algebraic structures, order structures, and topological structures. (Piaget, 1955, pp. 14–15)

Piaget's identification of Bourbaki's mother structures with the basic structures of thinking, implying a harmony between the structures of "contemporary" mathematics and the way in which a child constructs mathematical knowledge, had a straightforward pedagogical implication: The learning of mathematics takes place through the mother structures of Bourbaki, the structures with which 20th-century mathematicians had founded and built their science. Correspondingly, Piaget (1955) asserted that "if the building of mathematics is based on 'structures,' which moreover correspond to the structures of intelligence, then it is on the gradual organization of these operational structures that the didactics of mathematics must be based" (p. 32). In other words: A model for the science of mathematics was promoted as a model for mathematics education. Some teachers who participated in the 1952 meeting in La Rochette immediately adapted their teaching practice to what they had learned (Félix, 1986).

Bourbaki's reconstruction of mathematics from a limited number of basic structures, connecting different branches of this science and underlining its fundamental unity, "supported" by Piaget's theory of cognitive development, would further stimulate the debate within CIEAEM. For the 1954 meeting in Oosterbeek (The Netherlands), the theme "The modern mathematics at school" was chosen. One of the questions concerned the adaptation of curricula "in the light of what we know about modern mathematics and the thinking of the child and the adolescent" (Lenger, 1954–1955, p. 58). According to Frédérique Lenger, it was primarily up to the teacher to introduce something of the spirit of modern mathematics into their teaching.

> Modern mathematics [...] is the result of an awareness of structures and the relationships between these structures. The thinking of the modern mathematician is relational. And it seems to me that relational mathematical thinking can be recreated by a child or adolescent if the teacher is aware of it and if he presents the appropriate situations. (Lenger, 1954–1955, p. 58)

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a footnote to that book chapter.

2.2. UNESCO enters the scene

In the mid-1950s, a systematic effort to map national developments in the teaching of mathematics at the secondary level was undertaken by UNESCO, in cooperation with the International Bureau of Education led by Piaget (UNESCO, 1956). In an extensive survey of secondary school mathematics, in which 62 countries participated, one of the questions reflected a growing interest in modern mathematics: "To what extent does the evolution of modern mathematics affect secondary education?" (p. 10). The review of the answers of the 62 countries stated:

> The question as to what precise extent modern developments in the mathematical field have affected the secondary teaching of mathematics was answered by some twenty of the countries. In some cases the reply states merely that those developments were taken account of in the formulation of the secondary mathematics syllabuses. In other cases details are given of definite modifications made or impending in those syllabuses, including the introduction of infinitesimal calculus, coordinate geometry, statistics, etc., and added stress on functions, vectors, the calculation of probability, differential and integral calculus, and applied mathematics. (pp. 26–27)

Mathematical structures were not included in this list. Willy Servais, who was a delegate for Belgium at the UNESCO conference in Geneva in July 1956, at which the results of the survey were discussed, did make explicit mention of modern mathematical structures in his report:

> To what extent can the more abstract [...] mathematical structures, discovered within classical mathematics and developed worldwide by today's mathematicians, have a beneficial impact on secondary education? This is a very recent question to which pioneers in many countries are seeking an answer. The results obtained so far hold the promise of a pedagogical innovation. (Servais, 1956–1957, p. 40)

In general, survey responses revealed an awareness of the need for changes in secondary school mathematics curricula, but little mention of implemented reforms. At the national level in Europe, discussion about a modernization of secondary mathematics curricula emerged from the mid-1950s, particularly in France and Belgium. From the end of the 1950s, some concrete reform initiatives were taken. We mention two of them, both pre-Royaumont. In 1956–1957, at the Sorbonne University in Paris, Choquet organized, in collaboration with the *Association des Professeurs de Mathématiques de l'Enseignement Public* [Association of Teachers of Mathematics in State Schools] and the *Société Mathématique de France* [French Mathematical Society], a series of 17 lectures by research mathematicians – Bourbakists or mathematicians inspired by Bourbaki – to initiate secondary school teachers in the fundamental structures of modern mathematics (Barbazo and Pombourcq, 2010; Félix, 2005). In August 1958, in the margin of the 12th CIEAEM meeting in Saint Andrews (Scotland, UK), the Belgians Frédérique Lenger and Willy Servais compiled the draft of a concrete program for the teaching of modern mathematics, that was subsequently tested in two schools during the 1958–1959 school year, arguably the first attempt to teach "modern mathematics" in Europe (De Bock and Vanpaemel, 2019).

3. The early American New Math movement

While the early modern mathematics movement in Europe was driven primarily by new developments in pure mathematics and a psycho-pedagogical discourse, New Math – a collective name for a number of more or less related curriculum projects in the US – was from the outset "a political history" (subtitle of Phillips, 2015). A breeding ground was World War II and the subsequent Cold War with the Soviet Union. As early as 1944, the National Council of Teachers of Mathematics (NCTM) had launched a "Commission on Post-War Plans" to plan and make detailed proposals for strengthening high school mathematics (ages 14–18) in the post-War era (Roberts, 2023). It was felt that mathematics education was in a sorry state, whereas society needed more mathematically skilled citizens, workers and engineers. In this vein, local projects flourished in the early and mid-1950s to improve (traditional) mathematics education and bridge the gap between school mathematics and university mathematics.

3.1. Reform initiatives in the early and mid-1950s

In several European countries, with France as a prototypical example, school curricula are nationally defined. In contrast, in the US, the federal government has no control of education. That control lies within the states who in turn often delegate this responsibility to local governments. Hence, there is no national mathematics curriculum in the US. Fehr (1954) clarifies:

> The curriculum is made by state committees, by local committees, and by committees appointed by national organizations. These curriculums are, for the most part, guides which teachers can follow but must not adhere to rigidly. (p. 551)

This explains why New Math in the US was not a coherent thing; the term represents multiple projects in which schools or groups of schools could join. These projects developed curricula and instructional materials, and provided teachers with broader support for their implementation.

The University of Illinois Committee on School Mathematics (UICSM) is recognized as the first New Math project in the US. The original impetus came from the College of Engineering of the University of Illinois, being convinced that to keep pace with the modern world, their students would have to learn more and better mathematics than before and learn it earlier in their training (Hayden, 1981). The College revised its curricula accordingly. The UICSM was established in 1951 to prepare high school students for the new university standards for mathematics. To that end, a new school curriculum for mathematics was developed, primarily to improve the teaching of traditional mathematics and to make transition from high school to university easier. New Math as it came to be understood after 1960, was initially not on the agenda (Vanpaemel and De Bock, 2019). The new-developed curriculum was not directed exclusively at prospective engineering students; in fact, UICSM was soon convinced that all capable students would benefit from it (Walmsley, 2003). Max Beberman, a charismatic personality with great mathematical, didactical and political talents, became the head of the project and one of the central figures of the New Math movement in the US (Roberts, 2023). The first classes based on an experimental UICSM curriculum were taught in the 1952–1953 school year at a secondary school affiliated with the University of Illinois.

In terms of content, the UICSM curriculum differed from traditional ones in two important ways: it strongly emphasized the use of unambiguous and precise language, and it required careful and thorough explanations of basic concepts (e.g., the concepts of number, variable, function, equation). Classic became the distinction made between a number and a numeral (a number is the abstract idea, a numeral is the written symbol for that idea; so "5" and "V" are different numerals representing the same number). To provide thorough explanations of basic concepts, UICSM turned to research mathematicians familiar with the foundations of mathematics. Involving research mathematicians in designing a high school curriculum was never considered before; UICSM was the first project to do this (Walmsley, 2003). However, when the UICSM began to involve research mathematicians, typical New Math topics became incorporated into their curriculum (e.g., sets, relations, number systems, numbers in bases other than 10, the laws of commutativity, associativity and distributivity, functions as sets of ordered pairs of a specific type, logic). Apparently, the consulted research mathematicians found these more abstract ideas appropriate for high school students.

In terms of pedagogy, UICSM advocated "discovery learning", meaning that content and goals are set by the school, but that students should have the opportunity to discover generalizations along the road (Kilpatrick, 2012). Beberman considered discovery methods the only way to truly understand mathematics. In sum, precision in language and discovery were seen by UICSM as the key elements to promote understanding and make mathematics enjoyable for all students (Walmsley, 2003). Although dissemination of the UICSM project remained relatively limited, it became a model and inspiration for other New Math projects (Hayden, 1981).

In addition to UICSM, groups of mathematicians and educators from several other institutions undertook similar projects in the 1950s to reform secondary school mathematics (Hayden, 1981; Kilpatrick, 2012; Roberts, 2023). Noteworthy is the University of Maryland Mathematics Project (UMMaP), which started in the fall of 1957 under the leadership of John Mayor, a PhD in mathematics (Roberts, 2023). UMMaP initially focused on the junior high school curriculum (grades 7–8, ages 12–15). In terms of content, UMMaP emphasized four issues (corresponding in part to UICSM emphases): use of simple but precise language, emphasis on understanding rather than on rote learning, integration of arithmetic and algebra, and mathematical structure (Hayden, 1981; Walmsley, 2003). Different from UICSM was that UMMaP involved psychologists in its project, primarily to investigate from what maturity level specific mathematical concepts could be learned appropriately. That research led to the inclusion of all algebra, normally taught in grade 9, in UMMaP's junior high school curriculum. However, UMMaP, like UICSM, had not spread widely in the US before a much bigger project came along (see the next section).

In 1955, apart from the ongoing projects, the US College Entrance Examination Board (CEEB), an organization developing and administering standardized examinations as part of the college admissions process, appointed a Commission on Mathematics to study the mathematics needed for entering university, to review the existing secondary school mathematics curriculum, and to make recommendations for its modernization and improvement (Roberts, 2023). The Commission, consisting of college and high school mathematics teachers and trainers of mathematics teachers, was headed by Princeton mathematician Albert Tucker. Kilpatrick (2012) explains why CEEB had created the Commission:

> The Board's examiners were concerned about a growing gulf between their examinations and the mathematics being taught in some college preparatory programs as well as about the low levels of mathematical understanding and poor attitudes toward mathematics on the part of many high school graduates. (p. 564)

The Commission's final report made specific mathematics content recommendations for college-bound students. It did not call for uprooting tradition; instead, it called "for teaching traditional topics from a modern point of view" (Hayden, 1981, p. 112). In algebra, for example, a shift from manipulative skills to understanding was advocated, as was an introduction to deductive reasoning and the incorporation of topics such as sets, inequalities, functions, and the commutative, associative, and distributive laws. For the second semester of grade 12, the Commission offered a number of options, including an introduction to modern abstract algebra or a course in probability and statistics. The Commission's final report was widely distributed and had a strong impact on mathematics education in the US from the late 1950s.

3.2. Sputnik and the founding of the School Mathematics Study Group

An external event brought momentum to the New Math movement: the launching of the first artificial satellite, Sputnik I, by the Soviet Union on October 4, 1957. It challenged Americans' belief that they were technologically superior to the Soviets, and led to accelerated interest in and widespread support for education in science and mathematics among the American public. In early 1958, the perceived shortage of mathematicians and a growing concern about the mathematical preparation of the workforce led to two conferences of mathematicians and the establishment of the School Mathematics Study Group (SMSG). SMSG would become the largest New Math project in the US. It was headquartered at Yale University and directed by mathematician Edward Begle, a Princeton PhD. Soon, SMSG would be massively funded by the National Science Foundation. SMSG was dissolved in 1972 when this funding ceased (Phillips, 2015).

SMSG's strategy was to propose a revised mathematics curriculum supplemented with sample material for classroom use (Hayden, 1981). That sample material took the form of textbooks and these were produced for all grades of high school. The textbooks were authored by writing groups that brought together high school teachers and college mathematicians from all over the country. SMSG's writing groups agreed to take the recommendations of CEEB's Commission on Mathematics as a starting point and could benefit from the experiences in earlier curriculum projects, both in terms of content choice and pedagogical approach. The produced textbooks were trialed and then supplied directly to the schools, outside the publishing industry. Because, in practice, a textbook often determines what is taught, SMSG had a major impact on the curriculum actually implemented in schools through this strategy.

According to Phillips (2015), SMSG not only took the opportunity to refocus the rigor and scope of the mathematics curriculum, but also its overall approach.

> SMSG's mathematicians took the charge to make the intellectual habits of American students more rigorous as an opportunity to introduce "modern" mathematics into the curriculum. They argued that developments over the first half of the century had fundamentally reformulated what is meant to do mathematics. [...] The curriculum project was their opportunity to inscribe this view of mathematics in millions of textbooks. (Phillips, 2015, p. 47)

This new view of mathematics was inspired by the work of the Bourbaki group in France, which considered "structure" to be the essence of the discipline. SMSG emphasized that mathematicians did not calculate, instead their job was "logical reasoning." The first problems in SMSG's junior high school series "involved logic puzzles that required not counting or measuring, but careful reasoning about particular sets of information" (Phillips, 2015, p. 54). A most telling example was SMSG's treatment of arithmetic, not a tool for calculation, but a means to discover basic properties of numbers and operations. The authors introduced modular arithmetic and arithmetic in bases other than 10 with the aim that students would recognize the similarities and differences between the structure of these new forms of arithmetic and the usual arithmetic calculations (Phillips, 2015).

Once the first textbooks for high school were completed, SMSG began to focus on primary education as well. Begle realized that problems associated with the design of a curriculum and the preparation of textbooks for primary schools need to incorporate psychological understandings about child development and concept-formation into the teaching of arithmetic (De Bock and Goemans, 2023). Psychological support for Begle and his team to extend their efforts to the primary level was provided by Jerome Bruner, a leading Harvard psychologist who was strongly influenced by "structuralism" and Piaget's research. Bruner's work suggests that any subject can be learned in some intellectually honest form to any child at any developmental stage, as long as instruction is organized appropriately (Bruner, 1960). It provided SMSG with a psychological justification for going ahead with the New Math reform at the primary level (Phillips, 2015).

4. The Royaumont Seminar: A turning point in the modernization of school mathematics

In the late 1950s, the "Organisation for European Economic Cooperation" (OEEC) entered the educational scene. OEEC was founded in 1948 to manage the substantial funds of the US-financed Marshall Plan for the post-War economic and industrial recovery of Western European countries. In June 1958, OEEC had established an Office for Scientific and Technical Personnel (OSTP) with the goal of "promoting international action to increase the supply and improve the quality of scientists and engineers in OEEC countries" (OEEC, 1961a, p. 4). From November 23 to December 4, 1959, OSTP organized at the *Cercle Culturel de Royaumont* in Asnières-sur-Oise (France) a seminar on "New thinking in School Mathematics." That an economic body is concerned with mathematics education is likely related to the then strong belief that mathematics can be a means to achieve economic growth and prosperity. Or as Willy Servais, secretary of the CIEAEM and being involved in organizing the seminar, wrote:

> Progress is a condition for the development of countries, as the interest shown in the promotion of mathematics by the Organisation for European Economic Cooperation underlines. As far as we are concerned, we need to train [...] students who will love mathematics to the point of wanting to teach it or even create it. Training enough mathematicians is an essential objective. If we achieve it, we will have the solution to other problems. (Servais, 1957–1958, p. 3)

The Royaumont Seminar turned out to be a milestone in the history of the modern mathematics/New Math reform movement. As Skovsmose (2009) observed: "After the Royaumont Seminar, modern mathematics education spread worldwide, and dominated a variety of curriculum reforms" (p. 332).

To understand the dynamics at the seminar, it is important to know who participated (and who did not). Each OEEC member country was "'invited to send three delegates: one outstanding mathematician, another a mathematics educator or person in charge of mathematics in the ministry of education, and a third an outstanding secondary school teacher of mathematics" (OEEC, 1961a, p. 7). There were 30 delegates from 16 European countries, and three more form Canada and the United States. In addition, 13 guest speakers were invited, including one educationalist, William Douglas Wall. Actually, the seminar was dominated by Bourbaki-oriented mathematicians; in particular, Jean Dieudonné's slogan "Euclid must go!" became part of the collective memory. According to these mathematicians, the basic model for modernizing school mathematics should be the academic discipline of (pure) mathematics. Calls for integrating new applications and modelling in reforming school mathematics were also voiced in Royaumont, especially by Anglo-Saxon participants, but these were less decisive for developments in the 1960s than the dominant structuralist proposals. Hans Freudenthal did not participate in the Seminar, a decision he would later call "a cardinal mistake" (La Bastide-van Gemert, 2015, p. 212). Freudenthal had underestimated the impact the Seminar would have.

The Seminar was chaired by Marshall Stone, president of the International Commission on Mathematical Instruction. In his opening address "Reform in School Mathematics." Stone pointed to the "dislocation" between secondary and university levels of mathematical instruction, as a result of the extraordinary growth of pure mathematics in modern times, and the increasing dependence of scientific thought upon advanced mathematical methods. He also addressed the introduction of modern mathematics into the secondary school curriculum, which would mean incorporating "a few subjects or topics of fairly recent origin" and the elimination of "dead, useless, outmoded or unimportant parts of mathematics, however hallowed by tradition" (OEEC, 1961a, pp. 16–17). Stone did not specify which new or old parts of mathematics he had in mind.

The second speaker, Jean Dieudonné, was more specific: The culprit was the "pure geometry taught more or less according to Euclid" hence his provocative outcry "Euclid must go!" According to Dieudonné, most of the topics in a Euclidean geometry course have "just as much relevance to what mathematicians (pure and applied) are doing today as magic squares or chess problems" (OEEC, 1961a, pp. 34–36). Instead of the old curriculum, he proposed to teach several new topics organized in a curriculum for students from age 14 to 17, roughly starting from "experimental" mathematics, concentrating on techniques and practical work, to a rigorous, axiomatic treatment of the two- and three-dimensional space. Dieudonné was certainly not opposed to an axiomatic approach to mathematics – he admired the achievements of the Greeks in mathematics – but he argued that the Euclidean *corpus* could now be reorganized on simple and sound foundations, and the importance of its content needed to be re-evaluated in the light of modern mathematics. The foundations to which Dieudonné alluded are the axioms of a two-dimensional vector space equipped with a scalar product.

An alternative to the traditional teaching of Euclid, but still within the Euclidean framework, was presented by Otto Botsch. His proposal, *Bewegungsgeometrie*, consisted in a dynamic approach to geometry instruction, already in use in more than half of the secondary schools in Germany. The underlying inspiration of this proposal was Felix Klein's Erlangen Program for geometry based on groups. According to Botsch, the study of geometry should be preceded by the study of physical objects, including paper-folding, drawing, cutting and pasting, and the making of geometrical ornaments; only in a later stage, one could move to the study of translations and rotations which can subsequently lead to a geometry of vectors and to the study of the properties of groups. Edwin Maxwell, however, who lectured on a new syllabus for calculus, was critical of the modern vector-based treatments of geometry as proposed at the Seminar:

> I feel that the premature introduction of vectors is a possible source of real confusion to the young. The economy of effort which they allow is, of course, very real; but that economy is one for the mature mind, rather than for the beginner. (OEEC, 1961a, p. 89)

The ambition to bring unity to the secondary school mathematics curriculum was another major theme at the Seminar. This was particularly evident in the treatment of arithmetic and algebra, discussed respectively by Gustave Choquet and Willy Servais. Choquet proceeded from the fact that modern mathematics tended increasingly to dissolve the boundaries between arithmetic, algebra, geometry, and calculus. This could be done through the study of underlying structures such as groups, rings, and fields. He proposed that addition and multiplication of natural numbers be introduced as early as primary school through the union of disjoint sets and the product of finite sets, respectively. He also suggested doing a few calculations in the binary, octad, or duo-decimal system. "There can be no doubt," he concluded, "that the basic concept of numeration can only be thoroughly understood by the pupil if he has studied several different systems" (OEEC, 1961a, p. 67). Choquet's proposal was followed by a proposal for a modern and coherent approach to algebra by Servais. According to Servais, the teaching of algebra should not be confined to operations with numbers or numerical variables. Modern algebra is the study of operational structures, irrespective of the nature of the objects covered by the operations. Although Servais clearly saw mathematics as a deductive science, he favored an active and exploratory approach to mathematics education. "Definitions will be given progressively to make pupils aware of what they have acquired. [The] properties of the algebra of sets should be discovered rather than expounded" (OEEC, 1961a, p. 69).

New uses of mathematics, for example, in the social sciences, and their implications for mathematics education were discussed by Albert Tucker. He distinguished between problems of disorganized complexity, involving numerous variables, and problems of organized complexity, involving a sizable number of interrelated factors. The first category asks for techniques from probability theory and statistical interference; the second requires, among other things, knowledge and use of matrix algebra. He illustrated the second category by a problem of linear programming, utilizing inequalities, intersections, graphic methods, and unique algebraic procedures for solving equations. According to Tucker, integration into all secondary school programs of these newer types of mathematics, in an appropriate form, is feasible and desirable. However, he acknowledged that an effort is needed to enhance teachers' knowledge of modern mathematics and its applications so they can teach the subject well. Tucker's plea for the integration of probability theory and statistics in all secondary school curricula was supported by Luke Bunt, who presented the outline of a syllabus on the subject as taught in a Dutch experiment for the human science streams of secondary schools. For Bunt, the problem of estimating some characteristics of a population based on the values of these characteristics in a sample should be the dominant objective of a secondary school statistics course.

Undoubtedly, the intervention by Edward Begle about problems of implementation captured the attention of most Europeans. The Americans were ahead in the implementation of a reform that was considered of national interest. So, the federal US government was willing to invest "very large sums of money every year" in Begle's SMSG, something the Europeans could only dream of. Begle believed "that it is *not* a question of experimenting to see whether [a considerable improvement in the teaching of sciences and mathematics] can be made – we are convinced that it can be" (OEEC, 1961a, pp. 98–99).

A final paper on implementation problems was presented by William Douglas Wall, director of the National Foundation for Education Research in the UK. Wall outlined some steps that educational research should look forward to, but first he admitted that:

> Research into the learning and teaching of arithmetic and mathematics has, on the whole, been short-term, scattered and piecemeal. For the most part, too, it has erred by examining only one or two aspects of the problem at the time – whereas education essentially concerns a complex of interrelated variables whose dynamic interaction is qualitatively and quantitatively different from its parts taken separately or additively. (OEEC, 1961a, p. 101)

But, Wall continued, results of psycho-pedagogical research are often ignored by education administrators, teacher training institutions and the schools themselves. "They prefer the cosy comfort of unverified opinion and rule of thumb to the dangers of objectively verifiable hypotheses" (OEEC, 1961a, p. 102). Wall called for a coordinated, long-term, and multidisciplinary research effort, but showed himself critical about the suggestions for change as presented at the Seminar:

> The suggestions for change made in this Seminar have a justification in reason and logic perhaps but we have no means of knowing whether they will be as successful or more than the old ways, unless into our reform we build from the outset means of objective study and evaluation of results. (OEEC, 1961a, p. 103)

At the end of the Seminar, a list of resolutions was unanimously adopted. These can be briefly summarized as:

1. There is an urgent need to adapt the teaching of geometry and algebra in schools to the enormous advances in modern mathematics.
2. Elementary probability should be recognized as an appropriate part of mathematics taught in secondary schools.
3. Competent teachers should be attracted and retained to the profession.
4. Mathematics instructions in secondary schools should be provided only by university graduates majoring in mathematics.
5. Each delegation should prepare a small bibliography in order to familiarize mathematics teachers with important publications on the topics discussed.
6. The OEEC should encourage experimentation with the proposals made.
7. The OEEC should establish a group of experts to prepare a detailed synopsis for modern secondary school mathematics (OEEC, 1961a).

Following the latter resolution, the OEEC formed a group of experts who met in Zagreb-Dubrovnik in August and September 1960. The group would reach a consensus on the introduction of set theory, algebra, analysis, probability theory, and statistics, but the outcome for geometry was only a vague compromise (OEEC, 1961b).

5. Conclusions

In Europe, at the meetings of the CIEAEM in the early 1950s, especially at the 1952 meeting in La Rochette, the seeds were sown for a structural approach to the teaching of mathematics, i.e., modern mathematics. Bourbaki offered the mathematical rationale and Piaget provided the psychological justification. In the subsequent years, CIEAEM would continue to play an important role in refining its modern view on mathematics education.

Around the same time, curriculum reform projects emerged in the US that would be called New Math. A first project was initiated by UICSM and the ideas were more or less along the same lines as those of CIEAEM in Europe. With the founding of SMSG in early 1958, New Math quickly became established in the US, while concrete initiatives in Europe were still small-scale and limited mainly to France and Belgium.

Although some European and American reformers may have met before the Royaumont Seminar, for example, at the 1954 or 1958 International Congress of Mathematicians (Barrantes and Ruiz, 1998), there were no systematic contacts between the two reform movements in the pre-Royaumont era. In all probability, US reformers did not even know of the existence of CIEAEM, and conversely, there are little or no indications that European reformers were aware of what happened in the US. At the Royaumont Seminar, a substantial exchange and discussion of ideas about the future of school mathematics took place between reformers from both sides of the Atlantic. It would result in a breakthrough of modern mathematics, first throughout Western Europe and later the rest of the world. At various international meetings organized after Royaumont (see, e.g., Furinghetti and Menghini, 2023), Europeans and Americans had several opportunities to meet again, but there was no direct influence of US curricula on those in continental Europe (or vice versa). Thus, the European modern mathematics movement was certainly not an "import product" of the American New Math. On the contrary, our findings provide evidence for Bob Moon's claim that "a 'wave' of development in the USA 'crossed over' to Europe, although it is oft repeated, may be too simplistic a picture [...] a pattern of 'parallel' innovation would be a more appropriate characterization" (Moon, 1986, pp. 46–47).

But how then can it be explained that similar reform movements emerged more or less simultaneously on both sides of the Atlantic? The post-World War II period offered a favorable setting. It was a period of optimism and belief in socioeconomic progress and in the positive role science could play in realizing that progress (see, e.g., Servais, 1957–1958). Moreover, structuralism, as an approach to mathematical science, became a dominant paradigm in the 1950s in various life sciences such as psychology, sociology, anthropology, and linguistics (Gispert, 2010). In the first part of this article, we clarified Bourbaki's direct role in the emergence of the European modern mathematics movement. But Bourbaki was also a critical factor in the New Math reform in the US. From the 1950s, Bourbaki's treatises became known in the US, as did its manifesto *The Architecture of Mathematics* (Bourbaki, 1950). These publications had a major influence on American research mathematicians; many of them favored the Bourbaki approach, including those involved in New Math projects. Probably the most outspoken proponent of Bourbaki in the US was Marshall Stone from Chicago, president of the International Commission on Mathematical Instruction in 1959–1962 and a central SMSG writer. For Stone, "modern" mathematics was truly "abstract" mathematics separated from the physical world. He argued:

> A modern mathematician would prefer the positive characterization of his subject as the study of general abstract systems, each one of which is an edifice built of specified abstract elements and structured by the presence of arbitrary but unambiguously specified relations among them. (Stone, 1961, p. 717)

In addition to research mathematicians, psychologists also played a central role on both sides of the Atlantic (De Bock and Goemans, 2023). We discussed the role of Piaget in the early modern mathematics debate in Europe. In the US, stage psychologists, working in the tradition of Piaget, were involved in UMMaP, and Bruner's advice was central to SMSG. The advice of psychologists generally led to certain topics being covered earlier in the curriculum than had been the case in the past.

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Dirk De Bock
KU Leuven
Faculty of Economics and Business
Chair of the Department of Economic Education
Warmoesberg 26, 1000 Brussels
Belgium
e-mail: dirk.debock@kuleuven.be