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Observe – program. Math as a bridge between everyday life and programming*

Abstract. The paper describes how mathematics can be used as a bridge between the observation of phenomena (experiments from physics, animal life or observation of everyday activities) and programming in activities with students. The most important part of the proposed student activity is the construction of a mathematical model based on a function that reflects reality well. A description of workshops carried out with secondary school students is presented.

STEAM and Learning by doing idea

STEM Education is an approach to learning that uses Science, Technology, Engineering and Mathematics as access points for guiding student inquiry, dialogue, and critical thinking. Using STEAM education results in students who take thoughtful risks, engage in experiential learning, persist in problem-solving, embrace collaboration, and work through the creative process. Students in STEM programs may have more experiential learning opportunities, but they are limited to only science, technology, engineering and math. This is connected with idea learning by doing. This idea means that we learn more when we actually "do" the activity. This theory has been expounded by philosopher John Dewey and pedagogue Paulo Freire (Freire, 1982). It's a hands-on approach to learning, meaning students must interact with their environment in order to adapt and learn. Dewey believed that the best way to achieve that was to create a practical curriculum that had relevance to students' lives and experiences. Researchers like John Sweller have shown that short-term memory is often where learning happens. Only after short-term memory processes the problem, it can arrive in long-term memory. But, if learning by doing comes too early, then we can't learn. We need a time before we can master the knowledge. Learning by doing activity after you've already gained

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some familiarity with the content. It works because the technique calls you to actively engage with the material and generate the knowledge yourself, bit by bit. The idea of learning by doing works very well during robotics classes. About the fact that using a computer can help you make the process of learning mathematics more natural than formal wrote Papert (Papert, 1993, 1994).

This article describes robotics classes for high school students where we use the concept of a function. Classes can be conducted using any robots. In these examples, we use LEGO® SPIKE robots (see Figure 1) and Scratch or Python language.

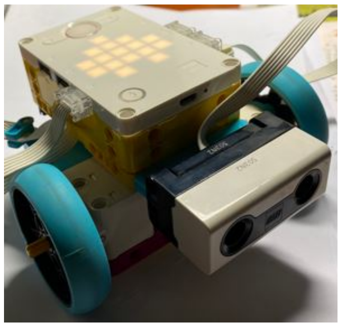

Figure 1: Model used during lesson

Simulation of cruise control and parking sensor using Lego Spike

Before students participate in the activities presented in this part of article, they have to be capable to solve systems of equations, understand the concept of functions and linear functions, and sketch graphs of functions.

Exercise: Let's look at the situation: A vehicle stops (or slows down). The following one should do the same and keep the safe distance between them. Write a function formula that can simulate the described phenomenon and program the robot to work as described.

At the beginning we are going to check the effective working range of the distance sensor and servomotor. The Distance Sensor measures distance to an object or surface using ultrasound. The sensor works by sending out high frequency sound waves that bounce off any object in range, and measuring how long it takes the sound to return to the sensor. It works with a range of 4 to 200 cm. Large angular motor is designed to function in models as both a motor and a sensor. With the integrated advanced Rotation Sensor, the motor can report both speed and position Speed Sensor (measures percentage of maximum designed speed – with the range -100 to 100).

The aim of the exercise is to find the function which would be capable of translating one set $[4, 200]$ into the other $[-100, 100]$. Let's consider the linear function $f(x) = ax + b$. Do we have any limitations? Let's look at the exercise again: the first vehicle stops – the following one should do the same and keep the safe distance between them (let's put 10 cm.) In that case we have $f(10) = 0$. It means that the vehicle will stop 10 cm before the obstacle. The second question is: when the second car should slow down. Let's assume that $f(50) = 100$. It means that the vehicle will start to reduce its speed 50 cm before the obstacle. By solving the system of equations students will receive the function $f(x) = 2.5 \cdot x - 25$ (see Figure 2 and Figure 3). Note that the obtained linear function describes the situation only in the interval $[4, 50]$. In the interval $(50, 200]$ we have a constant function equal to 100. This is due to the fact that when the velocity has a value greater than 100 (or less than -100), then the program automatically takes 100 (or -100 respectively). In the next stages of the lesson, students should sketch a graph of this function (some use the Wolfram Alpha program, others prefer GeoGebra, still others draw it themselves). They should mark the name of the axis (distance, speed) and mark the domain, the range, the point where the function is equal zero, the interval where the function is positive or it is negative (see Figure 4). After that students write the code in Python or Scratch language to see the simulation. They prepare the program that allows them to verify their expectations (see Figure 5 and Figure 7). Students should interpret the obtained graph of function and refer it to the behavior of the robot. This interpretation is the most important element of the lesson.

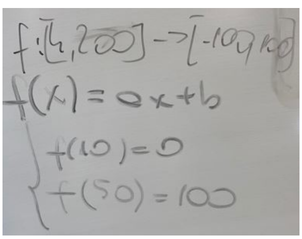

Figure 2: System of equations

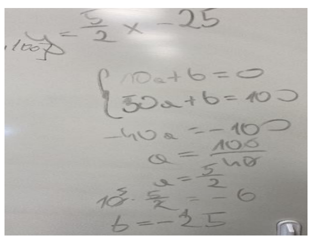

Figure 3: The solution of system of equations

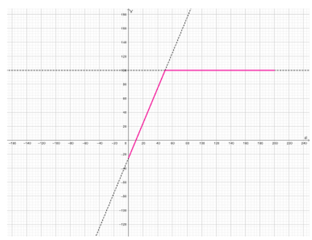

Figure 4: Graph of function in GeoGebra

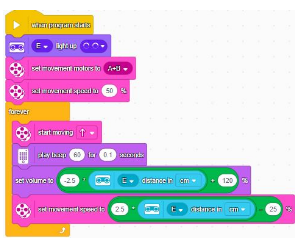

Figure 5: Program in Scratch

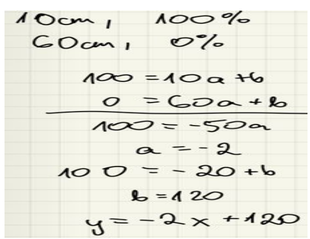

Figure 6: The solution: The closer the robot is to the obstacle, the louder the sound signal will be

Figure 7: Program in Python: The closer the robot is to the obstacle, the louder the sound signal will be

Sometimes students suggest modification. One of many examples is changing the second condition (i.e. f(50)=100). They recalculate everything for the new situation and compare the robot's behavior. The example can be extended to simulate a system of parking sensors mounted in vehicles. (In this model the more suitable name is: the detector of obstacles). Functioning method is simple. The closer robot gets to the obstacle the louder it will beep. We are constructing a linear function $g : [4, 200] \to [0, 100]$. The set of range depending on the working range of the speaker, where 0 means that there is no sound and 100 means it operates on the full power. The most common limitation suggested is $g(4) = 100$, $g(200) = 0$. This limitations don't represent reality and in this place the observation is an important starting point. The robot should start informing the user about the obstacle (by beeping) from the moment it starts to reduce its speed. So according to the example presented above it should be $g(50) = 0$. In Figure 6 and Figure 7 we present the situation where Agata proposes putting $g(10) = 100$, $g(60) = 0$. In this example function f was given by the formula $f(x) = 2x - 20$ (limitations was $f(60) = 100$ and $f(10) = 0$). During the lesson we often find ourselves referring to observation. The details play a vital role.

Circular motion

In physics, circular motion is a movement of an object along the circumference of a circle or rotation along a circular path. This very simple concept from a programming point of view allows you to combine physics, math and coding lessons. The idea was presented in (Sobera, Szczerba-Zubek, 2015). During one of the workshop we noticed an excellent opportunity for expanding the topic and use the rational functions. Before the activities presented in this part of article, students should understand the concept of rational functions, asymptotes, velocity and speed.

Exercise. Write a program that makes the robot move around a circle of given radius R (see Figure 8). Write a function formula connecting speed of wheel and radius R.

One of the differences between speed and velocity is that velocity can be zero or negative. It is because velocity is a vector and depends on the direction of movement. Negative velocity is a velocity in the opposite direction to the direction of movement that is considered positive. We will determine the linear velocity the right and the left wheels of the robot $v_1 = \frac{s_1}{t}$, $v_2 = \frac{s_2}{t}$. It is easy to see that we obtain $\frac{s_1}{v_1} = \frac{s_2}{v_2}$ and $v_1 = \frac{s_1 \cdot v_2}{s_2}$. The robot moves around the circle, so $v_1 = \frac{2 \cdot \pi \cdot r \cdot v_2}{2 \cdot \pi \cdot R}$ where r and R are the radii of the circles defined by the left and right wheel respectively. Putting $r = R - d$, where d means the distance between the wheels of the robot (marked in yellow in Figure 8) we have $v_1 = \frac{(R-d) \cdot v_2}{R}$. Let's write the resulting formula as a function of the variable R

$$v_1 = f(R) = \frac{(R - d) \cdot v_2}{R} = \frac{R \cdot v_2 - d \cdot v_2}{R}$$

We have obtained rational function. Its graph we present in Figure 9.

Now we can carry out an analysis of the correlation between the velocity of the right wheel and the left wheel depending on the sign of the difference $R - d$.

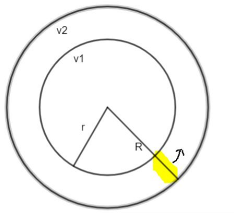

Figure 8: An illustrative drawing of the problem – the movement of the robot in a circle

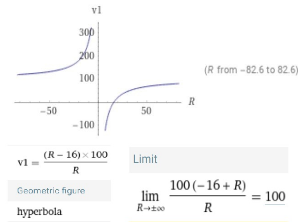

Figure 9: The solution: The rational function

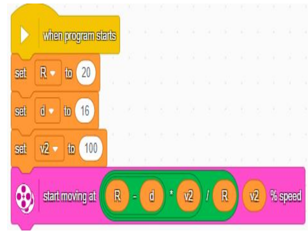

Figure 10: Program in Scratch

Students' task is to prepare the graph of the function determining asymptotes, domain, range, when the function is positive, when it is negative. Students should write a program in Scratch or in Python (see Figure10). And as was stated above students should interpret the obtained graph of function and refer it to the behavior of the robot (see Table 1). Moreover students should test the robot's behavior when $R > d$, $R = d$ and $\frac{d}{2} \leq R < d$.

$R-d$	v_2	v_1
$+$	$+$	$+$
0	arbitrary	0
$-$	$+$	$-$

Table 1: Correlation between the speed of the right wheel and the left wheel depending on the sign of the difference $R - d$

Conclusion

The knowledge and skills acquired in mathematics lessons were used in practice. Young people participate very actively in the proposed classes and after them they do not ask why they should learn mathematics. They saw that the basic skill in this topic was math skills.

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