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Realistic Answers in Open Word Problems: A Comparative Study Among Prospective Teachers*

Abstract. This study delves into the realism of answers provided by first- and fourth-year prospective mathematics teachers ($N = 115$) and prospective primary school teachers ($N = 126$) in an open mathematics word problem aiming to assess the longitudinal impact of two types of teacher education programs. Specifically, it assesses how the candidates' major and year of study influence their ability to provide realistic answers. Additionally, the study investigates the relationship between perceived proficiency in word problems and the quality of responses. The findings allow for inference on how the general professional preparation influences the sensitivity to solve open word problems in the case of prospective teachers who will teach mathematics in primary or secondary schools.

1. Introduction

In mathematics education, word problems can range from traditional exercises to real-life modeling problems. According to (Verschaffel et al., 2020), word problems can be viewed as a specific–often simplified–type of mathematical modeling problem:

> ... mathematical modeling tasks can be put on a "reality" or "authenticity" continuum, with traditional word problems constituting the negative pole, Kaiser (2017) modelling problems forming the positive pole, and various kinds of problem situations requiring the use of mathematics in between (p. 2).

To sum up, the research shows the difference between traditional word problems, which are usually simple exercises meant to help students practice specific skills,

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and real-life mathematical modeling tasks, which need students to understand complex situations, make assumptions, and use various mathematical tools. In this study, we use the term "open word problems" to indicate tasks that include more realistic elements and multiple solution pathways along different assumptions, thereby moving away from the purely traditional format but not necessarily reaching the full complexity of real-life modeling.

Understanding the above distinctions provides a comprehensive view of the challenges and complexities involved in the problem-solving process of word problems, particularly concerning the openness of mathematical tasks. Our study focuses on these two primary constructs within the context of word problems: open problems and realistic answers to these problems. Open problems are often characterized by their allowance for multiple answers or solution paths, fostering critical thinking among learners. On the other hand, realistic answers emphasize applying mathematical concepts to real-life situations, ensuring that solutions are contextually relevant. In the following paragraphs, we provide a detailed explanation of these constructs.

When scholars address open tasks, they typically refer to various constructs: some may envision authentic, real-world tasks, whereas others may consider the answer to be open or refer to open methods to solve the problem. Our work draws on the framework proposed by Yeo (2017), which characterizes openness using five task variables: goal, method, task complexity, answer, and extension. The first variable, goal, differentiates between tasks with a predetermined solution (closed goal) and those with multiple possible outcomes (open goal). The second variable, method, describes whether the approach to solving the task is well-defined with specific steps or is more flexible. The third variable, task complexity, refers to the level of difficulty or intricacy involved, which can be inherent to the task or subjective, depending on the solver's perspective. The fourth variable, answer, specifies if there is a single correct response (closed answer) or various valid responses (open answer). The final variable, extension, considers the potential for further exploration or elaboration beyond the initial task, which can be either an inherent aspect of the task or influenced by the solver's engagement and perspective. These variables provide a comprehensive framework for understanding and classifying the openness of mathematical tasks.

Realistic mathematical word problems have been integral to mathematics education for centuries. They are designed to practice basic arithmetic operations and prepare students to apply formal mathematical knowledge to real-life situations. However, the wording of the word problems is not necessarily real. They range from meaningless text to the real situation of modeling tasks (Galbraith and Stillman, 2001). Our study uses the concept of horizontal mathematization to define realistic mathematical word problems (Treffers, 1993). It emphasizes the use of mathematical tools to solve everyday-life problems. In this context, reflection becomes crucial, as it involves translations between reality and mathematics (Jupri and Drijvers, 2016). Some word problems, although set in a real-life context, are closed because they have exactly one correct numerical answer (Becker and Shimada, 1997). Because of this, solving them typically involves straightforward computations based on the data provided in the text, with little need for deeper modeling or interpretation. However, some word problems formulated in a real-life context can be considered open because they allow for multiple interpretations, which are not always immediately apparent to problem solvers. Often, these problems are "disguised as traditional problems" (Peled and Balacheff, 2011, p. 308), giving the impression that a straightforward procedure based on numerical data in the text will be sufficient for the solution. However, achieving a satisfactory solution requires a closer analysis of the context and the application of practical considerations. Failing to recognize the open nature of the situation can lead to incomplete answers.

The relationship between mathematical problem situations and real life remains unrecognized by many learners (Ambrus et al., 2019; Bonotto, 2002; Verschaffel, 2004; Verschaffel et al., 1994, 2002; Wyndhamn and Säljö, 1997). Studies have shown that even after several years of schooling, many students have constructed an approach to word problems so that the activity is reduced practically to executing one or more arithmetic operations with the numbers in the text and that real-life knowledge is not activated during problem-solving. This exclusion of realistic considerations and suspension of sense-making is particularly evident in word problem-solving in school mathematics. For example, Bonotto (2002) found that students often do not incorporate everyday experiences into their solutions, thereby missing opportunities to connect classroom mathematics with authentic contexts. The function of training students to apply formal math knowledge to real-world situations has been largely underutilized (Fitzpatrick et al., 2019).

A possible reason behind the unsatisfactory results concerning the realistic answers for open problems with real context is the "didactic contract" (Brousseau, 2002), which may include the obligation to answer the word problems in the way that is directly expected. This contract has been found to have a significant impact on students' abilities to solve realistic word problems (Jiménez and Ramos, 2011; Peled and Balacheff, 2011; Varga, 1988). The contract outlines the implicit and explicit rules of communication between teachers and students, which can either facilitate or hinder the incorporation of real-world knowledge into problem-solving processes (Delacour, 2016). While simpler, more traditional word problems undoubtedly play a valuable role in helping students build foundational mathematical skills and procedures, overreliance on them can inadvertently limit students' ability to apply real-world knowledge once the tasks become more open.

Research results on whether specific training can improve the situation are ambiguous. Some studies show that the situation does not necessarily improve with specific training on open problems but improves due to general mathematical literacy, and practice in mathematical problem-solving. Fitzpatrick et al. (2019) argue that, despite efforts to increase realistic responses through interventions such as response sentences, examples, and enhanced problem versions, these have largely been ineffective. Ambrus et al. (2019) present evidence that the relative frequency of realistic reactions in open word problems increases with students' grade levels, reflecting their expanding mathematical knowledge. Research by Kovács and Kónya (2019) also confirmed that even without special training, seventh-graders considered experts in mathematical problem-solving were more successful in solving open word problems than their peers with good mathematical skills but were considered novices. In contrast, research results for 9th-grade and 11th-grade students show that some improvement can be achieved with short-term, targeted training (Ambrus, 2020).

Given these unsatisfactory outcomes, our research focuses on teacher education: specifically, to what extent prospective teachers can realistically deal with the problems and whether they develop this disposition by the end of teacher training. The importance of these questions are crucial because, as Thompson (1992) argues, teachers' beliefs about the role of real-world knowledge in solving school arithmetic word problems significantly affect their teaching behavior and students' learning outcomes. In addition, Verschaffel et al. (1997) found that prospective teachers often overlook practical knowledge and practical considerations when solving arithmetic word problems.

In the present paper, we follow the approach of Verschaffel et al. (1994, 1997). In the 1994 study, the authors examined realistic responses to open problems among fifth graders. Later, in 1997, they repeated the research design with prospective teachers, comparing their performance to that of the younger students. The comparison revealed that while the prospective teachers produced a higher proportion of realistic responses than elementary school pupils, the overall percentage was still only 48%, which they considered "disappointingly low." This comparison highlights the persistence of the tendency to exclude real-world knowledge in mathematical problem-solving across different groups of learners. Our aim is to better understand this phenomenon by structuring the group of prospective teachers according to their future teacher qualifications (primary school teacher and mathematics teacher) and their progress in teacher training (those who have completed one year of study versus those in their fourth year).

Based on this rationale, our paper is guided by the following research question: To what extent are prospective teachers able to realistically answer disguised open word problems, and how do their responses vary based on their future qualifications (primary school teacher vs. mathematics teacher) and the duration of their training?

2. Methodology

2.1. Participants

This research was conducted in 2017 with students from two universities in Hungary. Prospective primary school teachers and mathematics teachers participated in the research. Primary school teachers in Hungary teach all subjects, including mathematics, in grades 1-4. Mathematics teachers teach mathematics in grades 5-12. Training for primary school teachers is eight semesters; for mathematics teachers, it was ten to twelve semesters for participating students. The survey was carried out in first grade and fourth grade. Detailed data are presented in Table 1.

Table 1: Major and grade of participants

	Major		
Grade	MT	PST	Total
1	54	47	101
4	61	79	140
Total	115	126	241

Note. MT for mathematics teacher and PST for primary school teacher

2.2. The task and coding of students' answers

Each survey participant was asked to solve the following word problem.

Since Pisti moved to a new house with his family, he has been receiving his pocket money of 1,000 Hungarian forints weekly. He has saved all of his pocket money since they moved. If Pisti has already saved 35,000 Hungarian forints, how many days have they spent in their new home? (Ambrus et al., 2019)

Note: "Pisti" in the problem is the common nickname for Stephan in the Hungarian language.

This task is open because there is a lack of information about the exact system for which Pisti receives the money, e.g., always on the same day of the week, on the seventh day, or irregularly. This realistic situation has to be modeled when answering. Considering the categorization by Yeo (2017) in this task, the Goal is closed in the sense that the answer should give the number of days. The Method is not uniquely determined, and the Answer is open because there are several possible answers. Complexity and Extension depend on the solver's approach. The task was solved individually during a lesson at the university, with twenty minutes allotted.

2.3. Coding system

Students' answers to the word problem were coded into three categories: Expected (EX), Realistic (RE), and Other (OTHER). Detailed descriptions and examples are provided in Table 2. The Expected and Realistic codes have been used as known from the literature (Ambrus et al., 2019; Verschaffel et al., 1994). The above sources use a detailed classification for responses other than Expected and Realistic ones. However, the frequency of these answers was low in our study (7%), so we grouped them into a single Other category.

In the appendix, we give some examples of realistic answers.

2.4. Students' questionnaire

A 5-point Likert scale questionnaire asked students about their proficiency in word problems (Q1, Q2), and whether all the data were given in the task (Q3) (see Table 3).

Table 2: The coding system with examples

Code	Description	Example
EX	The "expected answer" arises from the predicted direct use of the arithmetic operations prompted by the problem description.	$35 \cdot 7 = 245$
RE	A "realistic answer" results from using real knowledge about the context. The realistic situation was at least partly considered.	$245 \pm 6,\ 239 - 251,$ $245 - 251$
OTHER	The code is "other answer" if another type of response is given: the student had a reading comprehension problem, i.e., they did not answer the question, made an arithmetical error, or gave a number for which no reasonable explanation could be found, and the student did not provide such an explanation either.	5000 HUF, 35, 345, 270

Table 3: The Likert-skale questionnaire

No	Question
Q1	I like solving mathematical word problems.
Q2	I can quickly understand and see through mathematical word problems.
Q3	Everything was given for an unambiguous solution to the "Pocket money" problem.

Note. Scale: 1 strongly disagree, 5 strongly agree.

3. Results and discussion

3.1. Perceived profiency in word problems in the groups

For the questions on perceived profiency (Q1 and Q2) the reliability was high, with Cronbach's $\alpha = 0.761$. The Mann-Whitney U test revealed a statistically significant difference between mathematics teachers ($M = 3.760,\ SD = 0.760$) and primary school teachers ($M = 3.317,\ SD = 0.999$) with $U = 9236.000,\ p < 0.001$. The coefficient of variation for primary school teachers (0.301) was notably higher than that for mathematics teachers (0.202), indicating a broader spread in the self-assessed ability to solve word problems. No significant difference was observed between the first- and fourth-grade students ($U = 7076.000,\ p = 0.992$. This indicates a consistent perception across different academic years, suggesting that the progression through the curriculum does not significantly alter the variables under investigation in our study. This trend remained consistent if we layered the sample by majors.

3.2. The students' answers

Among the 241 responses, most were classified as EX, accounting for 63.9% of the total responses (Table 4). This indicates a significant inclination toward this category among the participants. The RE answers constitute 29.0% of the responses, highlighting a notable but less dominant preference than the EX category. However, this value is still much higher than among school-age pupils for this task, where the presence of the RE category was below 5% (Ambrus et al., 2019). The OTHER category, which encompasses responses that do not fit neatly into the EX or RE categories, represents a smaller fraction, with 7.1% of the responses.

Table 4: Frequencies for answers

Answer	Frequency	Relative frequency
EX	154	0.639
RE	70	0.290
OTHER	17	0.071
Total	241	1.000

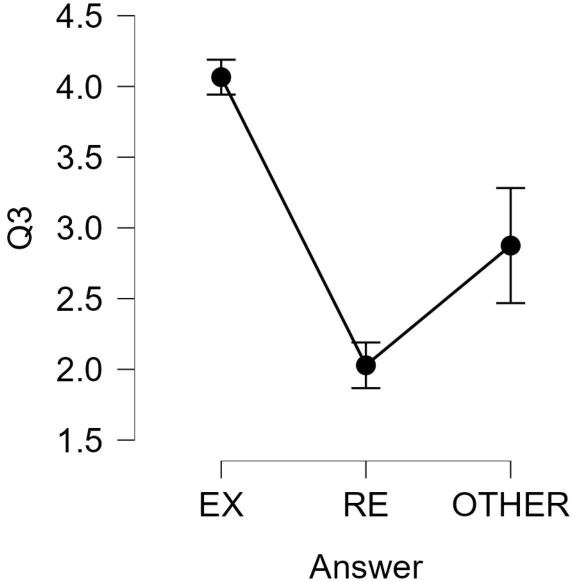
The answer to question Q3 (i.e., how openly the student perceived the problem) aligns closely with students' responses. Namely, those who perceived the task as open (low Q3 score) tended to give a realistic response, while those who perceived it as closed (high Q3 score) tended to give the expected response. This claim is supported by the non-parametric Kruskal-Wallis test that indicates a statistically significant difference in response distributions ($H(2) = 67.759,\ p < .001$), and Dunn's post hoc test revealed a highly significant difference between the EX and RE categories ($z = 8.127,\ p < 0.001$), as illustrated in Figure 1.

We consider this result remarkable because the perceived openness aligns with the type of response, indicating that students' understanding of the task's openness influences their answers despite any potential contradictory didactic contract about expected answers. The complexity of the educational interactions shapes students' responses not only by their understanding of the task's openness but also by their perceptions of the teacher's expectations. However, in the researched sample, the students' responses suggest a genuine alignment between perceived task openness and their problem-solving approaches.

In our research, the perceived proficiency in solving word problems was a reliable predictor of handling open word problems in a realistic context. The Kruskal-Wallis test indicates a statistically significant difference in response distributions ($H(2) = 19.012,\ p < .001$), and Dunn's post hoc test revealed a highly significant difference between the EX and RE categories ($z = 3.829,\ p < 0.001$), as shown in Figure 2.

These findings suggest that students who perceive themselves as proficient in solving word problems are more likely to provide realistic answers. This alignment between self-assessment and actual performance indicates that students generally have an accurate understanding of their problem-solving abilities. This self-

Figure 1: Distribution of perceived openness scores by responses



Note. The error bar is based on the standard error.

awareness can positively impact their approach to learning and tackling complex problems.

3.3. Analysis of student responses by major

The response patterns based on the students' major were analyzed using the contingency table in Table 5.

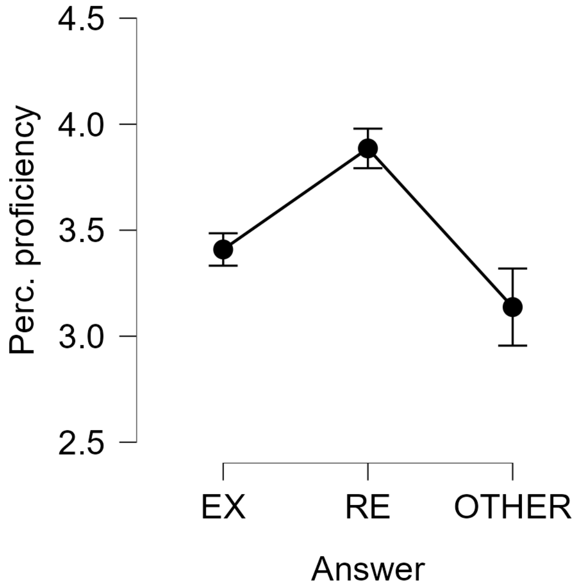
Table 5: Answers across majors

	Answer			
	EX	RE	OTHER	Total
MT	49	60	6	115
PST	105	10	11	126
Total	154	70	17	241

Note. MT for prospective mathematics teachers; PST for prospective primary school teachers.

The distribution of answers across different categories (EX, RE, and OTHER) revealed significant differences between the two groups. The result of the Chi-Squared test ($\chi^2(2)$=57.166, $p < .001$) indicated a statistically significant difference between the major and the type of response given. Prospective mathematics teachers were more likely to provide realistic answers ($N = 60$, 52.2%) compared to primary school teachers ($N = 10$, 7.9%); in contrast, primary school teachers

Figure 2: Distribution of perceived proficiency scores by responses



Note. The error bar is based on the standard error.

were more inclined towards expected answer responses ($N = 105$). Cramer's V value (0.487) suggests a moderate association between the major and response type.

The consistency across academic years underscores the robustness of the relationship between students' majors and response patterns to the "pocket money problem." The significant differences observed in the first and fourth years (see Table 6) suggest that the major field of study has a persistent influence on students' problem perceptions.

Table 6: Chi-squared test comparing MT and PST responses, layered by grades

Grade		Value	df	p
1	χ^2	19.744	2	< 0.001
	N	101		
4	χ^2	47.399	2	< 0.001
	N	140		
Total	χ^2	57.166	2	< 0.001
	N	241		

The findings confirm the hypothesis that general mathematical literacy, which is presumed to be higher among prospective mathematics teachers compared to prospective primary school teachers, significantly predicts the likelihood of realistic responses. This conclusion was reached even though the mathematical complexity of the tasks involved in this study was within the expected competence of prospective primary school teachers and merely necessitated arithmetic reasoning. The results highlight the importance of mathematical training in developing problem-solving approaches that consider real-world context.

3.4. Analysis of student responses by grade

Another aspect of our analysis is how the quality of the answers to the "pocket money" problem depends on the grade. Table 7 presents the distribution of responses across two grade levels.

Table 7: Answers layered by grades

	Answer			
	EX	RE	OTHER	Total
1	74	20	7	101
4	80	50	10	140
Total	154	70	17	241

The overall distribution indicates a shift towards more realistic responses as students progress from the first to the fourth year, suggesting a possible development in their ability to provide realistic answers over time. The increased number of realistic responses from the first year ($N = 20$) to the fourth year ($N = 50$) highlights this trend. The Chi-Squared test showed a significant difference in patterns ($\chi^2(2)$=7.506, $p = 0.023$). However, it was not so pronounced, and the Cramer's V value of 0.176 for the total sample points to a small association across all participants. When we layer participants by major, we find no significant change for primary school teachers ($\chi^2(2)$=5.858, $p = 0.053$), underscoring the combined effect of major and grade level on response patterns. However, for prospective mathematics teachers, the improvement is significant ($\chi^2(2)$=12.824, $p = 0.002$).

These results support our hypothesis that evolving mathematical knowledge significantly impacts response quality. While pedagogical content knowledge expands from the first to the fourth year for both majors, the increase in higher mathematics content is more pronounced for prospective mathematics teachers. In contrast, for prospective primary school teachers, the emphasis remains on content relevant to direct school practice, which may explain the lack of significant improvement in realistic responses for this group.

4. Conclusion

The key findings of the study regarding the realism of answers provided by first- and fourth-year students are as follows: First, evolving mathematical knowledge seems a basic predictor of providing realistic answers to real-life word problems, as supported by our result when layered the sample by grades or majors:

1. A significantly higher proportion of fourth-year students gave realistic answers than first-year students;

2. Prospective mathematics teachers were more likely to provide realistic answers, while prospective primary school teachers tended to give expected answers.

Second, there was no contradiction between perceived openness and students' answers. Respondents who recognized the task's openness were more likely to provide realistic responses. This finding strengthens (Krawitz et al., 2018) result that recognizing missing information is a major barrier to solving problems where the missing information is not obvious.

In summary, this study suggests that the development of competencies in mathematics teaching, as indicated by the grade, perceived proficiency in word problems, and the recognition of a task's openness, are key predictors of realistic responses in open mathematics tasks. This study underscores the importance of developing realistic problem-solving approaches among teacher candidates to improve mathematics education. The gap between school results and teacher candidates' results should be the subject of further research, i.e., what is the possible reason for the substantial difference between the results obtained in training and school practice?

5. Limitation

It is important to note that a limitation of our research is that our conclusions are based on the analysis of only one word problem.

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Appendix

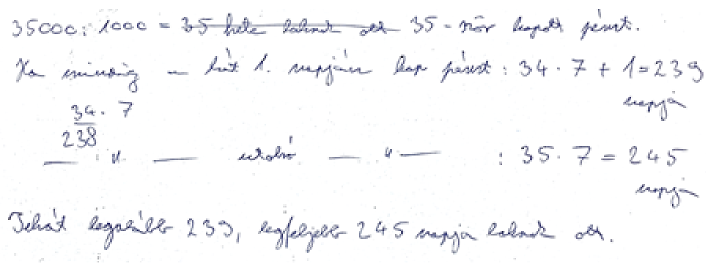

Figure 3: Realistic answer: 239–245 days

In Figure 3, we provide the first example of a realistic answer. The translation of the text is as follows:

> 35000 : 100 = 35 [Strikethrough: they have lived there for 35 weeks.] He received pocket money 35 times. If he always gets his pocket money on the first day of the week, he has lived there for $34 \cdot 7 + 1 = 239$ days. If he always gets his pocket money on the last day of the week, he has lived there for $35 \cdot 7 = 245$ days. So he lived there for at least 239 days and, at most, 245 days.

The student initially considered the problem closed but only got 35 weeks; see the crossed-out text. While solving the task, they realized that this only gave them information about the fact that Pisti had received the money 35 times. Later, the student supposed that the moving-in day was the first day of the week, and the first day of the week exactly coincided with the pocket money distribution day. Until the day of the 35th received pocket money, $34 \cdot 7 = 238$ days pass. However, the moving-in day is the first, resulting in 239 days. On the other side, 245 is the expected answer; however, this student explained it. The student considers the problem's openness, if not the fact that the payment day is not necessarily the day of moving in.

The second example is in Figure 4. The translation of the text is as follows:

> They move on Monday and get paid on Monday: 35 weeks $= 35 \cdot 7 = 210 + 35 = 245$ days. They've been [living there] for 245 days. [Strikethrough: they move on Tuesday and get paid on Monday. But if they get paid on Tuesday.]
> Tuesday they move: 244
> Wednesday they move: 243
> Thursday they move: 242
> Friday they move: 241
> Saturday they move: 240
> Sunday they move: 239.
> They live there for a minimum of 239 days and a maximum of $245+6 = 251$ days if they are paid at the end of the week instead of at the beginning.

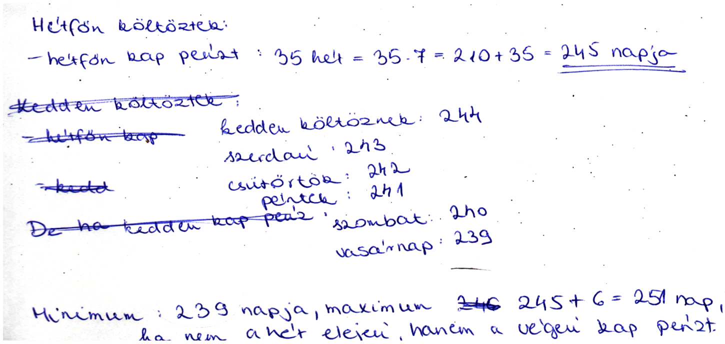

Figure 4: Realistic answer: 239–251 days

The student's hesitation is also evident in this work through the strikethroughs. The 251 days as a result is not explained by the student, but the thought process may be based on the fact that up to six days may have passed since the last payment. The values less than 245 days could be due to the fact that Pisti was paid for the week on Monday, but moved on Tuesday,. . . , Sunday.

In our last example (see Figure 5), the student adds to the expected answer (245 days) that a few days may pass since the last payment and that Pisti still has only 35000 HUF. Although they do not give an exact model for the task, they notice the openness and give an example that 248 days could have passed. Translation of the text:

> 35000 : 1000 = 35 weeks, $35 \cdot 7 = 245$ days. Consequently, they live there for 245 days.
> BUT! The reason they lived there for 248 days, for example, is that they moved on Mondays, and Pisti received money on Mondays. On day 248, he still only has 35000 HUF; he has [similarly] on day 251.
> Therefore, they live there for at least 245 days.

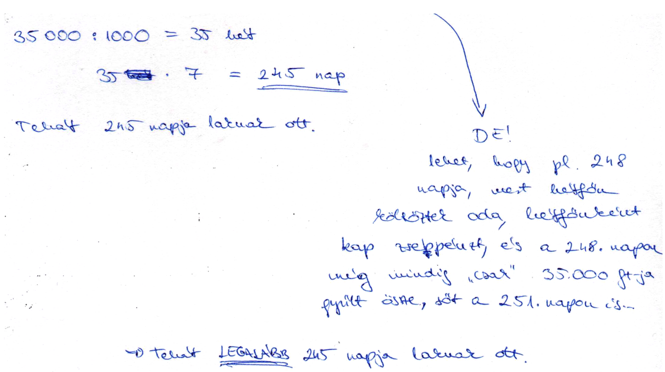

Figure 5: Realistic answer: at least 245 days

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