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Insights into Exponentiation: Ninth-Graders' Struggles and Teacher's Influence*

Abstract. Our research focuses on the development of ninth-grade student's concept of exponentiation. We conducted semi-structured interviews with four students to see to what extent they acquired the exponentiation concept by the end of the year. They had to solve four tasks, in which we could see which knowledge elements belonged to their actual level of development and which belonged to their zone of proximal development. Also, we were curious about the importance of the teacher's role as a guide in the process of concept acquisition. That is why we analyzed the students' and teacher's reactions. We found several reasons why it can be challenging for students to expand their exponentiation concept to negative exponents correctly and in the long term. Also, we can state that teacher's prompts can be determining factors in expanding a mathematical concept.

Introduction

A question that often arises during mathematics teaching is the process of concept acquisition. Teachers aim for their students to get as accurate a picture of the newly learned mathematical concept as possible. However, this can only be achieved if they establish first a sufficiently robust basis for the given concept. This is also the case when teaching the concept of exponentiation. When discussing exponentiation, the Hungarian Core Curriculum uses the spiral structure of mathematics. The development of the students' concept of exponentiation starts in elementary school and ends at the end of the secondary school years. During this long process, we add one new element of knowledge every year to what is learned so far. However, during the process, we automatically assume that the student's mathematical knowledge, which we can build, is sufficiently strong. Specifically, this means that to build the concept of exponentiation properly, students must

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also master many other mathematical concepts. (Sfard, 1991) From our previous research (Bihari, 2021) and further international research (PittaPantazi et al., 2007; Ulusoy, 2019; Baharuddin et al., 2021; Cangelosi et al., 2013; Denbel, 2019; Syafiqoh et al., 2018; Weber 2002), we know what obstacles we may encounter during this process and which types of errors indicate these obstacles.

In addition to developing students' mathematical concepts, examining what the teacher can do to build up the concept properly is worthwhile. It does matter what example the teacher uses to introduce and expand the given concept. It does matter how they connect it to the existing knowledge, just as how they check the students' concept image. Furthermore, the sometimes unplanned teacher reactions (prompts, hints, questions) can also be determining factors when solving a task.

In this article we present a preliminary study, which could be the first step of a long-term research related to concept development. This study analyses the results of four students' semistructured interviews according to two main aspects.

RQ1: What characterizes the students' actual and potential next level of development concerning the concept of exponentiation?

RQ2: How can teachers' prompts help extend students' initial concepts?

First, we examine the development of the concept of exponentiation, focusing on our students' actual and proximal development zones (Vygotsky, 1987). Therefore, RQ1 focuses on the answers of the students and further information suggested by their answers. With their verbal explanations and justifications, we could assess which knowledge elements are part of the students' actual development levels and which fall into their zone of proximal development. This will give us a better insight into the concept image that students have in their minds about exponentiation.

Second, we examine the role of the teacher in the concept formation process. RQ2 focuses on students' communication and meta-communication and mainly on the helpful prompts given by the teacher and their effects on the students.

In the theoretical section, we review the main issues of the concept formation process, particularly the exponential concept. After that, we describe the method of our research. Finally, we present the results of the interviews and draw detailed conclusions.

Theoretical background

The process of concept formation

Researchers dealing with the topic of concept acquisition have developed several theories that can be used to examine this process. However, some points seem to be common ground in all approaches. On the one hand, researchers, including Skemp (1976), all highlight the dual nature of understanding concepts. Skemp distinguished the instrumental and the relational understanding. Instrumental understanding means that students can learn certain rules and procedures without understanding how the concept works. The other type of understanding is relational understanding, when students understand the operation of the concepts and their relationship with previously learned concepts. Skemp said that these two types of understanding can be identified as "two different subjects being taught under the same name, 'mathematics'." (Skemp, 1976, p. 6)

On the other hand, the researchers all highlight the importance of building concepts on top of each other. According to Anna Sfard's (1991) model, insufficient understanding of previously learned concepts can seriously hinder the acquisition of new concepts. In mastering a concept, the student must go through three essential steps. These steps are interiorization, condensation, and reification. The new concept can only be put on a solid base if the students have reached the level of reification during the preliminary concepts related to it. (Figure 1)

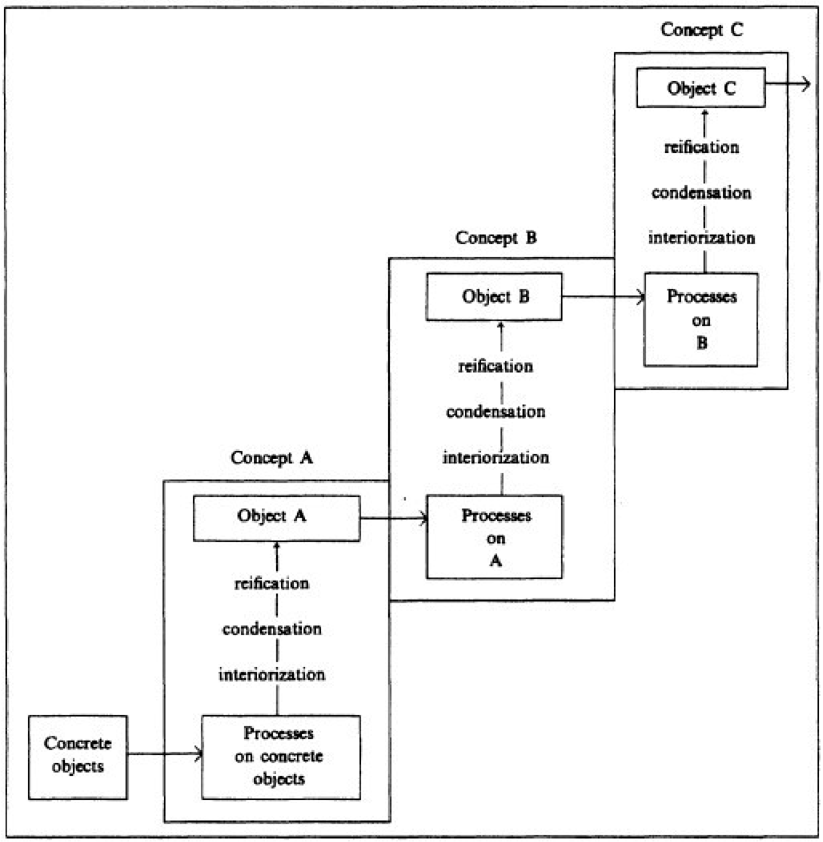

Figure 1: General model of concept formation (Sfard, 1991, p. 24)

The way to a deeper understanding of the concepts is not always easy for students. It usually involves significant energy investment. According to Hattie (2016), students learn in two ways. Some students practice specific algorithms and memorize rules without really understanding how they work. This is the surface learning. Students like this type of learning because they can achieve good results quickly with a small investment of energy. However, the result of this type of learning is not incorporated into long-term memory. On the contrary, during deep learning, when the students look for the answer "to the why," the understood procedures, rules, and concepts are recorded in long-term memory. (Hattie, Donoghue, 2016)

Development of concepts from the perspective of the student and the teacher

In mathematics classes, the teachers measure how deeply the students understand the newly learned concept through written tests. However, it usually does not give a complete picture of students' concept image. The assessments can only measure the actual level of students' development. According to Vygotsky (1987), to get a complete insight into our students' thinking, we must also examine their zone of proximal development. This zone shows the level the student can reach, not independently, but with a helping question or guidance. If the student can reach the zone of proximal development for a given task several times, this zone is moved up, and they can solve such tasks independently. At this point, we can see the vital role of the teacher's culture of questions in the process of concept acquisition. (Vygotsky, 1987)

When the student cannot do something independently, it rarely helps if the teacher presents the solution process again in a frontal way. However, at the right moment, the teacher's reactions to the student (helpful questions, hints, etc.) can push the student to reach their zone of proximal development. The teachers' prompts were categorized by Giles and Gilbert (1981). On the one hand, they highlight that prompts are more effective teacher-student interactions than pure frontal teacher explanations. Teacher prompts are aimed at three main goals. There are teacher reactions that aim to motivate the student during the problem-solving process (motivation-orientational prompts). It happens that the teachers want to draw students' attention to some rules they know, which are necessary to solve the task correctly (process-orientational prompts). In addition, the students may be unable to independently call up a critical rule or algorithm, which is indispensable for solving the task. In such cases, the teacher tries to re-evoke or reaware the student of the necessary previous knowledge with a question or a guiding statement (product-orientational prompts). (Giles, Gilbert, 1981)

In summary, the student and the teacher are responsible for understanding the concept. The teachers must respond to students' manifestations by developing their own questioning culture, asking deliberate questions, or even spontaneously promoting students' understanding.

The development of the concept of exponentiation

One crucial research on developing the exponentiation concept is Pitta-Pantazi's (2007) study. She highlights that the answers reveal what kind of prototype concept the students have and how much development this prototype leaves room for. By the prototype concept, we mean the first-seen examples and tasks the students can associate when they face an expression written in exponentiation form. Based on experimental research, Pitta-Pantazi categorized the students into three groups. The first group (low achievers) includes students whose concept of exponents is almost exclusively limited to the range of positive integer numbers (both the base and the exponent). When they worked on comparison problems with negative integers or fractions, they used the false analogy (misconception) obtained from their prototype concept, i.e., the larger number always has a larger exponent. Students in the next group (average achievers) could produce correct results with either a negative integer base or a negative integer exponent. They interpret exponentiation with a negative exponent not only instrumentally but also relationally, thanks to their more advanced concept of fractions. However, in the case of powers with fractional exponents, they also often encountered difficulties. The students of the third group (high achievers) succeeded in generalizing the concept of exponentiation for any kind of exponent. The relationship between the n^{th} root and the concept of exponentiation was sufficiently deepened, which indicated the successful accommodation of the existing prototype concept.

Our previous research also examined the concept of exponentiation among ninth-grade students. (Bihari, 2021) After analyzing the surveys, we identified several types of errors. Most students made errors based on a false analogy (i.e. 2^{-3} = -8). The students failed to sufficiently accommodate their existing prototype concept for the new rule to be assimilated correctly and in the long term (Mosonyi, 1972). We concluded that most students did not achieve the goal set in the Hungarian Core Curriculum; that is, the students did not master the concept of exponentiation with a negative integer exponent by the end of the school year. Therefore, most of them can be classified in the low achievers category, and only a few students are in the average achievers category. (Pitta-Pantazi et al., 2007)

Our survey results are supported by several international research on understanding the concept of exponentiation. The result of Turkish research (Ulusoy, 2019) shows that the type errors found - the same as those in our survey – stem from an insufficient understanding of the definition of exponentiation. In addition, the additive and multiplicative structure of the concept of integer numbers is underdeveloped for several students, which makes them unable to perform operations with powers correctly. At this point, the need arises to examine previously learned concepts essential for mastering the concept of exponentiation.

An Indonesian study (Baharuddin et al., 2021) examined the integer number concept of 13–15year-old students. According to the research, most procedural problems occurred in expressions containing negative numbers (negative sign). Furthermore, two studies, one conducted in the US (Cangelosi et al., 2013) and the other among Ethiopian university students (Denbel, 2019), also concluded that the negative sign and exponentiation with a negative exponent cause problems even among students in higher education.

Considering the abovementioned research, we can say that exponentiation can cause "worldwide difficulties" in teaching mathematics. That is why several researchers wanted to gain a deeper understanding of the students' thinking, so they conducted semi-structured interviews. Two high-achieving eleventh-grade students were interviewed during Indonesian research (Syafiqoh et al., 2018). Researchers found that students could perform operations in specific and general (algebraic) examples; however, they could not provide a general proof of any law of exponents. Weber's interviews with university students also show that a deeper understanding of students' concept of exponentiation is lacking (Weber, 2002). Most students see exponentiation only as a "mechanical rule," so they cannot understand its connections with other mathematical concepts.

Because of all these emerging problems, we decided to interview four of the students in our study mentioned above to see how deeply they understand the concept of exponentiation. Knowing the performance they achieved individually, we were curious to see what level they could reach with the help of a teacher (Vygotsky, 1987). Therefore, we analyze the interviews from the perspective of both the students and the teacher (Giles, Gilbert, 1981).

Method

In the school year 2019–2020, the performance of nine-grader students in three groups was followed in terms of exponentiation (Bihari, 2021). We did not intervene in the teaching process, but we had them take three assessments: a pre-test, a post-test, and a delayed test. A total of 48 students wrote all three. We conducted semi-structured interviews with four students after the delayed test. To select the interview subjects, the students were divided into two categories regarding the results of four delayed test tasks (Table 1). Students who solved all of them correctly fell into one category. Students who gave at least one incorrect answer were placed in the other. We randomly selected two students from both categories. Table 1 shows the students' S1, S2, S3, and S4 answers regarding similar tasks on the three tests (grey = error; white = correct).

Students	**Pre-test**				**Post-test**				**Delayed test**			
	3^4	$(-5)^3$	9^0	2^{-3}	3^4	$(-5)^3$	9^0	2^{-3}	2^5	$(-2)^3$	$\frac{2}{3}^0$	2^{-3}
S1	1242	No answer	1	-16	81	-125	1	-8	32	-8	1	-8
S2	81	-75	1	No answer	$\frac{1}{81}$	$\frac{1}{-125}$	1	$\frac{1}{8}$	32	-8	1	-8
S3	81	-125	1	8	81	-125	1	$\frac{1}{8}$	32	-8	1	$\frac{1}{8}$
S4	81	-125	1	-8	81	-125	1	-8	32	-8	1	$\frac{1}{8}$

Table 1: Individual answers to some test items.

The above-mentioned type error based on false analogy was found to be permanent for S1 and S2 in the delayed test. For S3 and S4, extending the exponential concept to negative integer exponents seems successful. The interviews were conducted via online video call at the end of the school year, on June 5, 2020. Each interview length was approximately 15 minutes. Four comparison tasks for the study were chosen from the delayed test, which was suitable for investigating four aspects of our students' exponentiation concept. During the discussion, the students were asked to argue for their answers. The interviews were videotaped with the consent of the students. We then made a transcript of each dialogue.

Result and Discussion

The results of the interviews are presented according to the exponentiation concept represented by four tasks.

Negative integer base, positive integer exponent

Task 1. Which expression is larger: $(-5)^2$ *or* $(-5)^3$?

Task 1 examines the relationship between the concepts of negative integer and exponentiation. Students should also be able to compare integer numbers correctly.

Based on the results, it can be stated that all the students successfully overcame the obstacle, i.e., they gave the correct answer to the question on their own, as shown in S1's explanation detail below:

S1:	(she thinks, looks at her paper) Well, first of all, I will calculate that $(-5)\cdot(-5)$ will be 25. And then after that, the same way, only three times. This will be a minus. That will be -125.
T:	Why is there a minus sign?
S1:	Since minus times minus is plus, I have three minuses here, so it will already be minus. So 25 is bigger. (She looks into the camera and explains her thought process to me. She smiles a little.)
T.	That's right, this is a good solution. Well done.

However, despite the correct answer, three students were hesitant about the truth of the following statement: "If the base of the power is a negative integer, then the exponent with the smaller positive integer is the larger". Only after the teacher questioned them did they realize that a counterexample could be used to refute the statement. Finding the counterexample was not a problem for any of them. In other words, the students could easily prove the falsity of the statement by the teacher's prompt, as shown in the following transcript.

T:	There [in the test] you indicated that this $(-5)^2$ is indeed the larger one, but the reason behind that was that „if the base is a negative number, then the smaller exponent is always the larger one." Actually, this is a wrong answer. Could you give some reason why the answer might be wrong?
S1:	Well, since it is not always the smaller exponent.
T:	Can you give a counterexample to this?
S1:	Well, let's say that $(-5)^4$ is the bigger one and not $(-5)^3$.
T:	Yes, it is a good justification. Okay, let's move on to the next task...

During the discussion, the teacher mainly used motivational prompts to confirm the correctness of the students' answers, increase their self-confidence (encouraging), and encourage the students to continue the solution (reinforcing). For example: "That's right, this is a good solution. Well done."

The teacher had to use process-orientational prompts (questioning) to draw students' attention to justify and generalize the solution, such as: "Could you give some reason?" and "Can you give a counterexample to this?"

Our experience is that all four students can raise negative integer numbers to positive integer exponents without much difficulty. They could easily refute their wrongly generalized statement based on a false analogy with little guidance from the teacher. That is, the student's initial prototype concept can be expanded with the help of a few process-orientational prompts, and wrong generalizations can be eliminated with examples that require only a slightly different thinking process regarding the known exponentiation.

Fraction base, positive integer exponent

Task 2. Which expression is larger: $\left(\frac{1}{5}\right)^2$ *or* $\left(\frac{1}{5}\right)^3$

The task examined the relationship between students' concepts of fractions and exponentiation. The interviews highlight that the fraction base was challenging for the students. S1, S2, and S4 could only solve the task with the teacher's help. S1 and even S4, who scored better in the previous tests, had difficulty while calculating the power value, as shown in the transcript below:

S4:	$\left(\frac{1}{5}\right)^3$ is greater because both numbers are fractions, and 3 is greater than 2.
T:	If we calculate, what will be the power value?
S4:	$\left(\frac{1}{5}\right)^2$ is $\frac{2}{10}$ and $\left(\frac{1}{5}\right)^2$ is $\frac{3}{15}$.
T:	How did you get $\frac{2}{10}$?
S4:	Oh... It's incorrect because I just multiplied them.
T:	Let's think again.
S4:	$\left(\frac{1}{15}\right)^2 = \frac{1}{5} \cdot \frac{1}{5}$
T:	That's right, that's what $\left(\frac{1}{5}\right)^2$ means.
S4:	Yes, and this is ...
T:	How do you multiply a fraction by a fraction?
S4:	Numerator by numerator, denominator by denominator. $1 \cdot 1 = 1$ and $5 \cdot 5 = 25$.

It can be read from the interview excerpt that S4 successfully revised his idea after a simple question, "How did you get?" and was able to multiply fractions correctly. After that, comparing fractions was no longer a problem for him.

S1 and S2, the students with lower abilities had the most difficulty comparing fractions besides performing the required operations. Both students needed more direct help from the teacher (Product-orientational prompts) to find the correct answer, as illustrated by the transcript below. This showed that the concept of fractions was not sufficiently mature; however, the comparison and multiplication of fractions can be classified in their zone of proximal development.

T:	When you learned fractions, you might have created examples like [...] I divide a cake into 25 parts ...
S2:	Yes, yes, yes ... (She nods.)
T:	Or into 125 parts. Based on this, could you explain which will be the smaller one?

S2:	Well, based on this, I say that $\frac{1}{25}$ is the smaller one. Since both are practically equal, I still think so. Nevertheless, $\frac{1}{25}$ is smaller since I divided it into 25 parts. (scared, turning her gaze downwards, straightening her hair)
T:	And the other one?
S2:	Well, 125 parts. (She says, looking at the camera more firmly.)
T:	And if I take 1 part out of the 25-part cake, will it be smaller than if I take 1 part out of the 125-part cake?
S2:	No. (She shakes her head quite firmly.)

The analogy given by the teacher brought the student closer to the correct way of thinking, as he was able to imagine what fractions mean. However, compared to the previous task, the student required much more specific help. The teacher did not lead the student to the correct thinking process (process-orientational prompt) but ultimately told her the path to the correct solution (product-orientational prompt). It is important to emphasize that even in this situation, the teacher tries to encourage the student (who lacks self-confidence) with motivational prompts, "No problem, you don't need to worry."

We can conclude that the three mentioned students have a less mature concept of fractions than integer numbers. The deficiencies can be partly classified in the student's zone of proximal development. In other words, their concept of fractions can be consolidated with the appropriate help of the teacher and practice. However, for the two weaker students, building a more solid concept of fractions required much more help since the teacher had to use more specific prompts (product-orientational prompts) to get the correct answer. The two students' current incomplete knowledge makes it even more challenging to extend exponentiation since powers with negative integer exponents also require a strong concept of fractions.

Negative integer exponent

Task 3. Which expression is larger: 3^{-2} *or* $\left(\frac{1}{3}\right)^2$?

The third task examined the concept of exponentiation by a negative integer exponent. We were curious whether the students had already successfully extended their concept of exponentiation or if it is at least part of their zone of proximal development, which means they can solve the task with the teacher's guidance.

Our first experience is that in the answers obtained in previous tests (Table 1), three out of four students made the same type of error when calculating the power value, as seen from the following quotations. (S1) "Um... (leaning a little right to left, looking at her notebook) Then it will be -9, won't it? (looking at her notebook shyly.)". (S2) "Well, right, 3^{-2} right, since there's a minus here, we have to put the 3 in the minus, I think ... (nodding, frowning, putting his hand in front of her chin) ... and $(-3) \cdot (-3)$ ... (looking into the camera, frowning) ... Well, it's 6. That is 9, sorry. (shaking her head, looking down)". (S4) "3^{-2} is bigger. ... Because it is -9 ... No, it is plus 9." S4, who performed best during the entire interview, only realized that he could not calculate the value of 3^{-2}. Therefore, it can be seen that none of the students could correctly and definitively extend the prototype concept of exponentiation by a negative integer.

The teacher, having assessed that the students could not find the right answer independently, wanted to guide them with supporting questions, i.e., product-orientational prompts. Using a juxtaposed example $\left(5^{-1} = \frac{1}{5}\right)$, the teacher tried to make the students aware of the rule of 5

exponentiation by a negative exponent. However, the application of the analogy was not easy. The experience of the interviews also showed that the students were not familiar with the concept of reciprocal, which is essential for a deep understanding of the extension. For example, S4 confused the concept of a reciprocal number with the opposite and the absolute value of a number.

T:	First, let's take the reciprocal of the base.
S4:	-3.
T:	It's the opposite.
S4:	To 3, it is 3.
T:	The reciprocal of 3 is $\frac{1}{3}$. You swap the numerator and the denominator.

The findings clearly showed that, despite the results of the previous surveys, none of the students could correctly connect the rule of exponentiation by a negative exponent to their existing scheme in the long term. Even after giving an analogous example, it was difficult for the students to complete the task. Compared to the previous examples, the role of the teacher was the most prominent here. That is, the teacher mostly took control of the solving process of this task and also had to correct and clarify the students' sentences several times, exploring the lack of previously learned concepts. For at least three students, the extension of exponentiation by a negative exponent is outside the zone of proximal development. In other words, students should strengthen the necessary fundamental concepts to learn the new knowledge effectively.

Raising to the zeroth power

Task 4: Which expression is larger: $(-3)^0$ *or* -3^0?

The last task aimed to examine the role of the 0^{th} power and the order of operations (the role of brackets).

S3 and S4, the two students with better abilities solved the task correctly alone or only with a few supporting (process-orientational) prompts. However, as seen in the following dialogue, the other two students needed more help.

S1:	Well, these two numbers are equal. (she said almost immediately as she described it)
T:	Why?
S1:	Because the zeroth power of every number is 1. (she says shyly, sometimes rolling her eyes)
T:	This is true. How is the exponentiation done here? In the case on the left, what exactly does the raising to the power of zero refer to?
S1:	Well, to 3 ... That is, to -3. (she corrected quickly, looking at the camera)

T: And for the other one, what does the raising to the zeroth power refer to?
S1: To the 3. (she says firmly, looking into the camera)
T: Why only to 3?
S1: Because we don't put... brackets there.? (she said questioningly, uncertainly, looking at her notebook)

Both students with lower abilities first repeated the well-learned rule and asserted that the two power values were equal to 1. This error is the so-called „sticky sign" type error (Cangelosi et al., 2013), considered a typical error in several experimental research mentioned earlier and our previous survey. However, some hesitation was felt in the students' answers; it means they perceived that the brackets have some role that cannot be ignored. After the teacher's prompts drew their attention to this („In the case on the left, what exactly does the raising to the power of zero refer to?"), they were first unable to correct their answers, so additional process-orientational prompts were needed to reach the correct answer.

In connection with the task, we were also curious to know at what level the students understood the rule of raising to the zeroth power. More specifically we wanted to see if they understand that this definition fits well into the conceptual system of exponentiation, and is not just a new rule out of thin air. For this reason, we asked them to argue that why the definition of the zeroth power is consistent with what they have learned so far about exponentiation. Students consistently answered, "Well, because there is this rule." After that, with the teacher's guiding questions and instructions, they understood the rule for a specific number. Based on these examples, we can say that the students know the stable but only mechanically learned rule for the zeroth power. Thus, they cannot always pay attention to the correct order of operations when applying it. However, this can be corrected with some teachers' prompts, i.e., it is part of the student's zone of proximal development.

Conclusion

Summarizing the experiences gained from the four semi-structured interviews, now, we answer the research questions of this preliminary study.

RQ1: What characterizes the students' actual and potential next level of development concerning the concept of exponentiation?

After analyzing the interviews, we can divide the experience of the four tasks into two groups. During the solution of Tasks 1 and 4, the common point is that the teacher used motivational and process-orientational prompts to encourage the students. As for the students, in both tasks, they could find the way to the correct solution by themselves (sometimes with a few prompts from the teacher). So, we can say that these students' actual level of development includes positive and negative integer-based exponentiation with positive integer exponents, as well as raising to the zeroth power, as a mechanically performed operation. In addition, their zone of proximal development includes comparing these types of powers and the correct handling of brackets and negative signs.

The other group consisted of Tasks 2 and 3, where the students encountered more enormous obstacles, mainly due to fractional numbers. The underdevelopment of their fractions concept causes extra difficulty when applying the mechanically learned exponentiation rule for negative integer exponent. In these cases, the teacher tried to re-aware the forgotten/missing knowledge with product-orientational prompts. However, three students still needed help answering the question correctly after giving the analogous example. Since the students required much more specific help in these tasks, exponentiation with a negative integer exponent does not belong to their zone of proximal development. They need to learn more about fractions, such as the concept of the reciprocal or how to compare fractions. Based on Sfard's theory and these experiences, we can conclude that the students' undeveloped concept of fractions is one of the fundamental deficiencies. So, extending their concept of exponents correctly and permanently to negative integer exponents is impossible.

RQ2: How can teachers' prompts help extend students' initial concepts?

As for the role of the teacher, we have seen that the teacher's questions and guidance, if chosen suitably, help the students to find or continue the right way of thinking. Just as when building mathematical concepts on top of each other, the principle of gradation is also essential when using teacher prompts. In other words, unless the student requests, the teacher should not use any assistance other than motivational prompts. In need of more specific help, the teacher must not miss a single step from the motivation–process–product chain, if possible. If the teacher recognizes these valuable moments during the task solution or afterward, they can know the development level of their student concerning the learned concept. The teacher can realize which knowledge belongs to their student's actual development level and which may be in the zone of proximal development. In the same way, the teacher can also determine if learning a particular concept means too big a step for the students. The teacher can evaluate which previously learned concepts need to be reinforced, providing the intermediate step for students to master or expand a concept.

We want to point out the limits of our work. Our preliminary study is not representative due to the small number of students participating in our survey and interviews. However, the results obtained are similar to the results of the studies mentioned above, and this suggests that it may be worth repeating the investigations with a more significant number of students in more grades to map the progress of the concept of exponentiation.

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