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Algebraic Approach Towards Shaping „Minus” in School Mathematics – AMMA project

Basic Information about the project

In September 2022, a project called “Algebraic Approach Towards Shaping “Minus” in School Mathematics” (ID 2022-1-PL01-KA210-SCH-000084007 with the acronym “AMMA”) won funding in the “Erasmus+ KA2 – Cooperation for innovation and exchange of good practices” competition in the category “KA210-SCH – Small-scale partnerships in school education”.

The project leader is the Pedagogical University of Krakow, and the project partners are two Slovakian and two Polish schools, namely:

1. Rudolf Dilong Primary School in Trstena (Základná škola s materskou školou Rudolfa Dilonga, www.zsdilong.sk) – mathematics teacher: Eva Kuníková, MA.
2. Primary School in Rabcice (Základná škola s materskou školou Rabčice, <http://www.zsrabcice.sk/>) – mathematics teacher: Ing. Lubová Beáta, PhD
3. Primary School no. 155 in Krakow (Szkoła Podstawowa nr 155 im. św. Jadwigi Królowej w Krakowie, www.sp155.krakow.pl) – mathematics teachers: Marzena Zięć, MA. and Mariola Czarnota, MA.
4. Independent Nobel Prize Winners Primary School in Krakow (Prywatna Szkoła Podstawowa z Oddziałami Dwujęzycznymi nr 2 im. Noblistów Polskich, <https://www.edukacja.krakow.pl/>) – mathematics teacher: Paulina Mołęda-Krupnik, MA.

Aim

The project concerns the improvement of mathematics education for primary school students (grades 5–8), including high ability students (grades 5 or even

lower) and low ability students (grades 7–8).

The project aims to develop, implement in schools and evaluate two sets of learning environments to shape

1. the concept of negative numbers and addition and subtraction of whole numbers
2. and the concept of opposite monomials and algebraic expressions and their addition and subtraction.

at the primary school level (grades 5–8).

The effective teaching of such an elementary concept as negative numbers and the elimination of difficulties in understanding algebraic expressions is the primary area of interest and fits closely with the statutory needs and areas of activity of all the institutions involved in the project.

Theoretical background and explanation of our approach

The theoretical mathematical basis of this approach is relatively new. Due to Artin-Schreier's account of ordered fields, we have distinguished two models of negative numbers: the first consists of an additive ordered group, while the second consists of an additive group and a set closed under addition. Thus, the first model assumes a total order of numbers, while the second a specific subset of numbers. The theoretical background, historical and pedagogical perspective includes the article by co-authors of the project (Błaszczuk, Sajka, 2017).

Pedagogical practice in Poland and Slovakia almost exclusively applies the first model. Through that approach, a number less than zero defines a negative number. As a result, in other stages of education, students commonly interpret “ $-x$ ” as a negative number rather than an element opposite to x . This misinterpretation leads to profound difficulties in understanding, interpreting, and processing algebraic expressions. Our approach, called “algebraic”, aims to bypass this misunderstanding and other obstacles as confusing x^2 with $2x$.

The proposed learning environments for the introduction of negative numbers with the use of an educational game are developed according to the constructivist (Lerman, 1989; Ertmer, Newby, 2013) and activity-based teaching approach (Krygowska, 1977), with the use of concrete countdowns (tokens), and then transferred to the teaching of opposite monomials and selected algebraic expressions, namely the addition and subtraction of second-degree polynomials (grades 7–8).

The algebraic interpretation of “minus” as “the opposite of” is consistent with “minus” for “ $-x$ ”. We seek to boost that approach through a simple game (and at some point in the future as a tablet or smartphone game) that may be suitable for students with special educational needs. The game is to assist students with difficulties in understanding negative numbers, operations on negative numbers, and then algebraic expressions in older classes (grade 8); it can also be suitable for inclusive education for students with various dysfunctions; furthermore, it can address mathematically gifted children as early as grade 4–5 of primary school or even earlier at the propaedeutic level.

Why is international cooperation necessary?

The main advantage of the international cooperation within the project is related to the different curriculum systems in Poland and Slovakia, as well as to the different socio-cultural and educational environments and finally to the different languages. As far as curricula are concerned, in Poland negative numbers are usually taught from grade 5, at the latest grade 6, while in Slovakia it is usually taught from grade 8. The different curricula allow us to assess first whether the effectiveness of our approach depends on the age of the pupils and the structure of the curriculum. The socio-cultural factor and the language factor are also important in the implementation and in the subjective perception and evaluation of the game by the pupils. This last factor may also depend on a difference between a public school and an elite private school, which are partners in this project.

Project tasks of the institutions

The role of the Pedagogical University of Krakow in this project is:

- 1) to develop the concept of teaching negative numbers using concrete models – tokens,
- 2) to prepare and conduct training for teachers on the concept of teaching negative numbers using concrete models,
- 3) to develop the concept of teaching algebraic expressions using concrete models – tokens,
- 4) to prepare and conduct training for teachers on the concept of teaching algebraic expressions using concrete models,
- 5) to evaluate the effectiveness of the concept of teaching negative numbers using concrete models (tokens) on the basis of empirical data,
- 6) to evaluate the effectiveness of the concept of teaching algebraic expressions using concrete models (tokens) – on the basis of empirical data.

The role of the partner organisations, as primary schools of different types (public and private), is to implement the learning environments and administer the written tests and questionnaires.

Current status of our work

Most of the project tasks and activities have been already completed. Learning environments have been developed and tokens have been designed for students and teachers (magnetic for the blackboard) for both the teaching of negative numbers and algebraic expressions. The tokens play a key role in the project, as they allow students to directly and individually manipulate models representing mathematical concepts, thus initiating a process of internalisation. The concept developed thus meets the requirements of constructivist teaching and the activity-based method. Teachers have been trained and are now implementing the learning environments developed.



Figure 1: Various activities of the AMMA project

The activities carried out so far in the AMMA project indicate that the project

- aims to modernise the teaching of concepts related to integers, algebraic expressions and operations on them,

- contributes to a better understanding of the concepts and operations on integers and algebraic expressions and to the development of students' mathematical thinking.

An additional advantage of the project is that it provides an opportunity to combine the research aspect with the training of pre-service teachers. Pre-service teachers of mathematics should have a background that enables them to carry out educational research and to identify and solve theoretical and practical problems related to the teaching of mathematics. The AMMA project provides an opportunity to meet this requirement. This is because 7 pre-service mathematics teachers have had the opportunity to participate in the project work. They had the opportunity to take an active part in selected training sessions, discussions about the research concept and research organisation. They had the opportunity to discuss with mathematics educators and primary school teachers – project partners from Poland and Slovakia. They also had the opportunity to participate in the research itself, to observe in practice students' reactions to the use of tokens in teaching selected topics related to integers. Some of them were observers or supporters of the experiment in one of the Polish partner schools.

Another important element of preparation for future teaching was the students' attempt to evaluate the effectiveness of the experiments carried out as part of the project. They carried out this work under the guidance of the project contractors from the Pedagogical University of Krakow, and the results of their activities are likely to form part of their diploma theses.

All the lessons conducted so far have received high initial ratings from both pupils and teachers in all schools in both countries. However, in-depth analysis and evaluation of these concepts will be the subject of further studies.

If our project is successful in the classroom, the next step will be to disseminate the concept and the game, prepare a tablet or smartphone game to facilitate the game within a wider range of partnerships in school education.

References

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