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Inspiring pre-service teachers of early years education to create an inquiry environment for children to learn mathematics*

Abstract. The paper presents a teaching proposal for the Mathematics Education course for university students of Education with a specialisation in Early Childhood Education, in order to motivate pre-service teachers to self-discovery and to encourage individual mathematical problem solving. The proposal was pre-tested in practice. Interactive activities were carried out in which pre-service teachers individually solved mathematical problems at thematic stations and had the opportunity to learn by manipulating objects and discovering solutions to problems. The study was conducted using an action research approach. The results of the study are described in four areas of the researcher's activity: preparing the learning environment (organising the research), observing the subjects, communicating with the subjects (interviewing the pre-service teachers) and using the results.

Motivating pre-service teachers of early years mathematics education

Every year for the past five years that I have been teaching *Mathematics Education* to pre-service teachers of Early Childhood Education, I have heard statements from course participants that express a dislike of mathematics, or a manifestation of their inability to solve a mathematical problem on their own, or a lack of motivation to learn. Pre-service teachers are concerned about the content and skills covered in the *Mathematics Education* course. They often emphasise that their knowledge of mathematics is only sufficient to solve tasks at the primary school level from grades 1 to 6, but that knowledge at higher levels of education already causes them great difficulties.

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From my observations in the first years of teaching pre-service teachers of Early Childhood Education, it was clear that they had great difficulties during the mathematics education course. Most of them did not make independent attempts to solve mathematical problems. The pre-service teachers were afraid of making mistakes in this process. They preferred to wait for ready-made solutions to problems. The passivity of the pre-service teachers later influenced the lack of non-schematic ideas for mathematical activities that would stimulate the children's cognitive curiosity.

The question arises whether pre-service teachers of early years education can be motivated to solve mathematical problems? And if so, how? The term motivation or motivating comes from the Latin word *movere* and means to move, to set in motion, to encourage someone to do something, to stimulate (Gasiul 2007, p. 222). According to Brophy, motivating pre-service teachers is about finding ways in which the teacher can encourage them to accept the goals of their work and to learn the knowledge and skills, regardless of whether they enjoy the activity and whether they would do it if they did not have to (Brophy 2007, p. 14). Petty distinguishes between long-term and short-term motivating factors, including: what I am learning is useful and the qualifications I gain from learning are needed; I am usually successful at school and this improves my self-esteem; what I am learning is interesting and arouses my curiosity; and learning activities are enjoyable (Petty, 2015, pp. 50–52).

Ideally, therefore, a pre-service teacher should be motivated to learn mathematics intrinsically, satisfying his or her natural curiosity and enjoyment of the mathematical world. However, this is not always possible. Therefore, a university teacher's teaching of mathematics should be interesting, innovative and encourage self-expression and creative thinking.

Taking into account the facts described above, I tried to create such conditions for pre-service teachers to learn mathematics, so that they could discover mathematics in a safe and enjoyable way, get to know it as an interesting field of knowledge that is useful in life and that, with some effort, can be understood and gives satisfaction in solving problems independently.

In constructing the research tool, i.e. the *thematic stations*, I tried to ensure that the mathematical problems presented were not boring or schematic school knowledge; furthermore, I selected the problem topics in such a way that they could be used by the pre-service teachers in their further studies or future work with children.

Research methodology

To improve the quality of education, action research can be used with the aim of changing educational practice (Czerepaniak-Walczak, 2010). The knowledge gained from the research is intended to enable a change in practice (Czerepaniak-Walczak, 2014, p. 325). The action research paradigm is characterised by the following features

- It is situational, i.e. it is concerned with diagnosing a problem in a specific context and trying to solve it in that context,

- The team members themselves are directly or indirectly involved in carrying out the research,
- it is self-evaluative, i.e. the modifications are continuously evaluated in real situations, as the aim is to improve practice,
- is a collective action, i.e. a team of researchers and practitioners (the same people are both researchers and practitioners) work together on a project (Czerepaniak-Walczak, 2014, p. 325).

The set of activities described in this article is part of a larger project. It describes the preparation and implementation of one of the mathematics education courses for pre-service teachers of early childhood education. Among other things, the results of these observations will be used to make changes in educational practice. The mathematics education lessons conducted also served as an inspiration for the work of the pre-service teachers – they were used by the study participants to conduct mathematics lessons in an interactive form for children in grades 1–3 in primary school.

As part of the research undertaken, I formulated the research question:

Does the designed interactive form of teaching motivate pre-service teachers of early years education to solve mathematical problems independently?

The subject of the research were the solutions to mathematical problems and the opinions of the pre-service teachers interviewed. The determinants of motivation were the number of pre-service teachers who tried to solve given problems independently, the correctness of these solutions, as well as the pre-service teachers' opinions about what they learned, what they managed to do, what interested them, what they would like to know more about a given topic, and what they paid attention to while dealing with a given problem. In the study I used the free observation method, school achievement tests (tool – worksheets available at each thematic station) and the diagnostic survey method (technique used – written categorised interview).

Research on the use of the interactive form has been conducted in three series of thematic stations. This paper proposes a third set of problems grouped in 5 thematic stations. The first two sets of thematic stations, which were solved by the same group of subjects, have been described in the articles: Bojarska-Sokolowska (2022a, 2022b).

The sessions were conducted for 92 pre-service early years teachers (53 from extramural studies, 29 from full-time studies) in smaller groups, so that each person could move freely through the prepared stations. Each session lasted 100 minutes for all groups. At each of the stations on a thematic problem, in addition to the objects to be manipulated, the use of the problem in real life was presented. This was intended to arouse curiosity about the given problem and ultimately increase the pre-service teachers' motivation to learn mathematics. In addition, each of the proposed problems could, with appropriate adaptation, be used to lead mathematics activities for children in grades 1–3.

Pre-service teachers could individually choose the position they wanted to work at any given time. They could work on it with other people or on their own. They

worked on a given problem for as long as they wanted. Below is a description of the stations in the third series of topics.

Description of the teaching proposal

In the first station – “Sudoku” (Fig. 1), the rules for filling in the numbers from 1 to 9 in a square diagram consisting of 9 squares with an area of 3 x 3 were explained. Once they had learnt the rules for solving the Sudoku, they had to write the numbers in the missing spaces (see: Moscovich, 2009).



Figure 1:
Thematic station 1 – Sudoku

		9			1	4	8	
			8		4	6		
8			2	6				
5	8	7				1		9
		2	7	1	3			
6	3				5		2	4
2	1			3			4	
		8	1	4				3
		4	6	7	2			

Figure 2:
Sudoku, Problem 1 (adapted from Guzicki, 2013, p. 21)

The other two problems were:

Problem 2. Explain why a one can be entered in the box marked with an asterisk (as the first digit entered in the Sudoku diagram).

	5	8	*			6	2	
			9		7			
7	1			2			4	9
		3	2		5	8		
6								1
		9	6		4	5		
3	6			9			5	4
			4		1			
	8	4				9	1	

Figure 3:
Sudoku, Problem 2 (adapted from Guzicki, 2013, p. 21)

Problem 3. Check and explain what number can be written in the box marked with an asterisk in the Sudoku diagram below.

7			5	8				
9	8			7			5	3
	5			4			8	6
8						3		7
	6	7	*	4	9			2
		2	7	6				8
4		5	2				3	
6				8		2		
	2		3			6	4	

Figure 4:
Sudoku, Problem 3 (adapted from Guzicki, 2013, p. 21)

At each station, additional information, history or curiosity about the theme was displayed. At thematic station one, the history of the Sudoku puzzle is described.

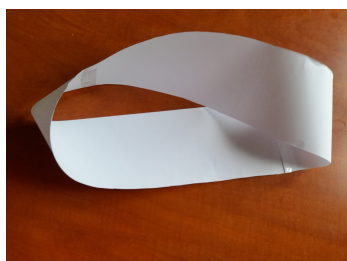
At station two entitled “Möbius Strip” (Fig. 5) pre-service teachers were provided with paper strips and scissors (see: Szarygin, Jerganzyjewa, 1995).



Figure 5: Thematic station 2 – Möbius Strip

The following is a description of the worksheet presented in this thematic station.

The exhibit in this station is called the Möbius strip and is made up as follows:



Take one of the strips and tape the ends together, after turning one of the ends 180 degrees.

Figure 6: Möbius strip

This strip has interesting properties. Draw a line with a pen without taking your hand off on the folded strip, what do you notice?.....

A description of the two experiments is then presented:

Experiment 1. *Assemble the seven strips one by one, increasing the number of turns before gluing them together, then study how many sides the resulting objects have and fill in the table. Number the subsequent strips, you will need them for the second experiment.*

Number of 180-degree turns before strip gluing	Number of sides of the page
0	
1	
2	
3	
4	
5	
6	

Experiment 2 Complete the table. Use the strips you made earlier by cutting them lengthwise with scissors.

Number of 180-degree turns before strip gluing	The result of cutting (what is formed)	Description
0		
1		
2		
3		
4		
5		
6		

Conclusion:.....

In the theoretical part, reference was made to topological objects, presenting the torus, the Klein bottle, among others. Other experiments that can be done with the Möbius strip were also presented.

In the third thematic station entitled “Cube Nets” (Fig. 7) the pre-service teachers were asked to find 11 cube nets.



Figure 7:
Thematic station 3 – Cube Nets

From the 35 hexominoes (plane figures formed by joining the edges of six squares, they cannot be connected only by vertices), select those that can be the nets of a cube.

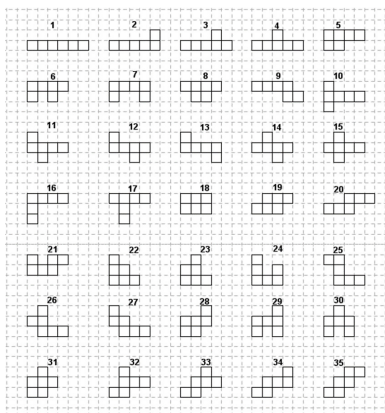


Figure 8:
Work sheet – Cube Nets (adapted from Ibanez, 2012, p.131)

In addition to information about hexominoes, the theoretical section contains information about pentominoes, i.e. different ways of connecting the sides of the

five squares.

In fourth thematic station “Fractals” (Figs. 9, 10), pre-service teachers were given three problems to solve.

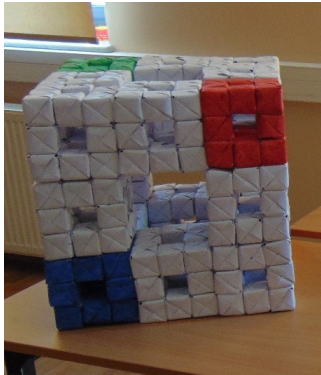


Figure 9:
Thematic station 4 – Fractals



Figure 10:
Some theory about fractals at station 4

The object shown in the photographs (Figs 9 and 10) was first described as follows:

A solid glued together from cubes made by the origami technique is a fractal, a self-similar object called a Menger cube. A Menger cube is formed as follows

Step 1. A cube is given.

Step 2. We cut it into 27 cubes of the same size, with planes parallel to the walls.

Step 3. We remove all the cubes adjacent to the centres of the walls of the original cube and the cube in the centre of the original cube.

Step 4. We apply the previous procedure to each of the 20 remaining cubes. After an infinite number of repetitions of the operations described, we obtain a Menger cube.

Then three problems are formulated, two of which are related to this fractal:

Problem 1.

Look at the glued fractal and write how many small cubes it is made up of? You need 6 modules to make a cube using the origami technique. How many modules were used to glue this fractal together?

Problem 2.

Each face of a Menger cube is also a fractal called a Sierpinski carpet. Describe the steps of the Sierpinski carpet construction.

Finally, Problem 3, concerning another fractal.

Problem 3. Complete the drawing of the fractal you started according to the instructions.

We have a square with a side length of 8 cm. We then build a square with a side length twice as long on each side (drawing). Repeat this construction for as long as possible. Paint the squares drawn in the same step with the same colour.

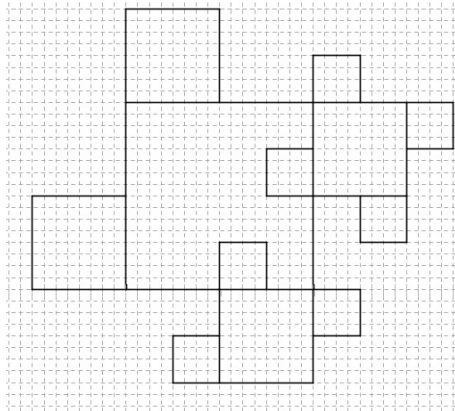


Figure 11:
Drawing of the fractal (own work)

In the theoretical part, pre-service teachers were able to learn about illustrations of other fractals, their structure and the history and application of fractals in human life.

In the fifth thematic station entitled “Squares” (Fig. 12) offered problems inspired by Hanisz’s (1997) items, modified for the study. There were sticks of equal length, which pre-service teachers used when solving the following problems:



Figure 12:
Thematic station 5 – Squares.

Problem 1. Take 20 sticks of the same length. Use the smallest number of sticks to make three squares. Draw them.

Problem 2. What is the largest number of sticks you can use to make three squares? Draw them.

Problem 3. How many sticks from 1 to 20 can you use to make three squares? Remember that they can be big or small. They can contain each other, they can have common sides, etc. Draw all the possibilities.

In the theoretical part, other stick puzzles were discussed, e.g. with rearranging sticks to get a given figure or figures.

Analysis of research results¹

The first thematic station was visited by 53% of all pre-service teachers. The Sudoku diagram proposed in Problem 1 was solved correctly by 94% of pre-service teachers. The correct rationale for Problems 2 and 3 was given by 81% of the pre-service teachers, who described why a one can be written in the given place in the Sudoku diagram. These same pre-service teachers correctly selected the number that can be written in the given space and justified this fact by eliminating other numbers.

From the written interviews with the pre-service teachers, it was clear that in the course of this thematic station they learned that “with small steps you can solve the task” (S21), “you have to use a strategy to be able to solve the Sudoku” (N12), “there is always a possibility of a solution despite the initial difficulties” (S4), “to prove why a number should have its place in a given box” (S7). Some pre-service teachers took a more emotional approach to the task, treating the solution of the problem as an achievement: “solving the Sudoku” (N37), “solving the first problem and explaining why 1 can be written in the asterisk and why 8 can be written in the asterisk” (N32). Respondents also mentioned what made them curious about solving the problems in this item: “getting to the solution” (S3), “how to write the numbers correctly” (S3), “the way the numbers are set up in Sudoku” (S24), “the different understanding of Sudoku by others” (N2). Some people mentioned that they would like to learn more about the topic: “about techniques to make solving Sudoku easier” (S5), “about more complex Sudoku” (S13), “about the quick way to get a solution” (N19), “about using Sudoku in everyday life” (S1). Both full-time and part-time pre-service teachers noted the amount of work it takes to solve a Sudoku: “I have to concentrate so that I don’t make a mistake when I enter the numbers” (N27), “you have to look at the numbers in a Sudoku with concentration” (N21), “accuracy and perceptiveness are the essence of solving it” (N14), “that Sudoku is an absorbing and long-lasting game” (S6), “that Sudoku can be relaxing” (N49), “that Sudoku requires thinking” (S23), “that this type of task is interesting and can be given to children to solve so that they develop their thinking” (N13), “that it can be an interesting way for children to get used to numbers” (N23). Some study participants stated that “Sudoku is fun” (S13), “playing with numbers is great” (S25).

The second thematic station was visited by 63% of all pre-service teachers. The characteristics of the strip were correctly identified by 83% of the pre-service teachers: “From one point you return to the same place” (S21), “I noticed that the line never breaks” (S1). Only 28% of the pre-service teachers correctly wrote the conclusion of the first experiment: “With an even number of turns there are two sides of the card. But with an odd number there is only one side of the card” (S12). More than half of the respondents doubted whether zero was an even number or not. Experiment 2 was solved correctly by 89% of the pre-service teachers, of whom 87.5% reached the correct conclusion. It was clear from the pre-service teachers’ statements that they had learned to “cut the strips and carry

¹In the next part of the paper, the symbol S with a number denotes the code of a full-time student, the symbol N with a number denotes the code of a non-full-time student. I have put the statements of the respondents in quotation marks and have not changed their form.

out the experiment" (S3), "describe the relationship between the rotations given" (S8). Some also mentioned what they found interesting: "that the strip does not disintegrate when cut" (S23), "this strip has very interesting properties" (N18), "the number of strips formed" (N35). There were also those who mentioned that they would like to learn more "about the use of the strip" (S14), "about the Möbius strip" (S18), "about strip tasks" (N2). Pre-service teachers noted that "the strip is an interesting material to work with" (N42), "how a new strip is formed when you cut it" (S14), "that the more turns we make and cut a piece of paper, the more strips are formed" (N12), "that this kind of task is very interesting" (N17), "that the strip resembles some everyday objects" (N33).

The third thematic station was visited by 64% of all pre-service teachers. 73% of them found eleven hexomino nets. The most common errors were in choosing the arrangement of the hexominoes numbered: 17 and 29, one person gave the possibility of 31. It was clear from the pre-service teachers' statements that working on this problem taught them to "arrange figures from nets" (N27). Some participants also mentioned what interested them when solving this item: "the form of the task" (S16), "the fact that I could manipulate the blocks" (N38). Respondents wrote that they were able to: "stimulate imagination" (N11), "arrange cubes from some nets" (S2). Pre-service teachers noted that: "if the bases are on one side, it is impossible to make a cube" (S15).

The fourth thematic station was visited by 62% of all pre-service teachers. 97% correctly calculated the number of cubes and modules used for the glued fractal. Correctly described the construction of the Sierpinski carpet – "We draw a square and divide it into 9 equal parts – squares; we remove the middle square. We divide each of the remaining eight squares into 9 equal parts-squares, remove the middle square, in the next steps we do the same" – 94% of the pre-service teachers. 32% of the pre-service teachers drew the fractal correctly according to the given instructions. From the pre-service teachers' statements it was clear that they had learnt while working in this position: "what a fractal is" (S16), "how a fractal is formed" (S29), "geometry has a number of relationships in it" (S28). Some people also mentioned what made them curious while solving the problems in this article: "how a fractal is built" (N26), "how to make a cube from origami" (S16), "how to start drawing a fractal" (S33). There were also people who mentioned that they would like to learn more: "about origami from fractals" (S11), "about fractals" (N50), "about the Menger cube" (N22), "about interesting geometric figures" (N37). Female pre-service teachers, both full-time and part-time, noted that "that the task was quite easy" (S14), "that you can make this cube indefinitely" (N21), "the connections between small and large geometric figures" (S8), "completing the fractal according to the instructions was not as difficult as I thought at the beginning" (N12).

The last thematic station was visited by 69% of all pre-service teachers. All of them found the smallest fold of 10 sticks, no one went lower than 9, 8 or even 6 sticks. The maximum fold of 20 sticks was found by 87% of the female pre-service teachers. Folding the sticks for any number between 10 and 20 sticks was found by 59% of the pre-service teachers. The pre-service teachers' statements indicated that "there are many combinations of arranging squares of sticks" (S7),

“just rearrange e.g. two sticks and something else comes out” (S33), “you can draw in many ways” (N1). Several people considered success as getting the task right: “arrange all the possibilities” (N11), “arrange 3 squares from different numbers of sticks” (S17), “think of many possibilities” (N43). Some study participants also mentioned what made them curious while solving the problems in this thematic station: “the fact that you can come up with an interesting task with sticks” (N25). There were also people who mentioned that they would like to know more: “how many combinations there are” (N51), “about stacking squares” (S25), “about variations with fewer sticks” (N19). Pre-service teachers, both full-time and part-time, noted that “this is a very interesting task” (N3), “you can use sticks in an interesting way” (S24), “this task can be adapted to different age groups” (N12), “that a stick can be the wall of two squares” (N31), “with the same number of sticks you can make different drawings” (S3). In addition to motivating pre-service teachers to try to solve mathematical problems, the form and themes of these activities were also intended to inspire female pre-service teachers to plan and carry out activities for children in primary school grades 1–3. And so it happened, with eight pre-service teachers using the inspiration of problem one to prepare activities for the children involving filling in a square sudoku, consisting of four 2 by 2 squares, as well as a square consisting of nine (3 by 3), sixteen (4 by 4), or twenty-five (5 by 5) squares. The pre-service teachers proposed not only filling these spaces with numbers, but also geometric shapes as well as other objects or animals.

Seven pre-service teachers offered the children a station where the children performed experiments with drawing and cutting the Möbius strip and ribbon and dividing the ribbon into 3 parts lengthwise, colouring the centre and cutting along. Five used yellow Reko blocks for the children to assemble cube nets. Three pre-service teachers offered a thematic station with drawing or cutting and gluing a model of Sierpinski carpet. Ten study participants suggested geometric and number problems for children to rearrange sticks, according to given rules.

Conclusions

The study found that the described interactive form of learning induced motivation in the majority of the surveyed pre-service teachers of early years education to attempt to solve mathematical problems. A greater “mobilisation” and “persistence” of the pre-service teachers surveyed to deal with mathematical problems was observed during the activities than during the traditional exercises. During the traditional form of teaching, usually only a few pre-service teachers tried to solve the problems and the others waited for the opportunity to write the solution on the blackboard or from a colleague at the desk. The pre-service teachers liked the fact that they could decide which problems to work on, in which order, for how long and with whom. Those who returned their worksheets received individual feedback. The pre-service teachers were still able to work on the problems they had not managed and wanted to complete after class. They were able to present their work in the next class where the individual tasks were discussed without being evaluated (only the proposal and implementation of interactive activities for

the pre-service teachers was evaluated). After the activities, the pre-service teachers described them as enjoyable and referred to their childhood memories, adding that mathematics was not so difficult then. In addition, it was clear from the pre-service teachers' statements that they were surprised that "maths is not just about counting" (S27). They liked the form of the lessons – "this form of exercises is attractive (N22)" and the fact that teaching aids were used during the lessons: "Mathematical tasks are much easier when we can use a model" (N48). After the surveys, it could be seen that the pre-service teachers' suggestions for activities for children were mostly more inspiring than before to formulate interesting research problems for children.

In summary, it could be said that motivating and inspiring pre-service teachers who have not so good memories of mathematics education should be based on providing a conducive learning environment; opportunities for interaction, use of collaborative, challenging learning in groups. (Krajewska, 2021, p.13). A key task for teachers is to create an overall climate of friendliness, "in which pre-service teachers have positive attitudes towards each other; structures and processes that meet pre-service teachers' needs and make pre-service teachers persevere in learning tasks and cooperate with each other; pre-service teachers' mastery of the social skills necessary to meet the cognitive and social demands of their environment" Arends (1998, p.122).

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