FOLIA 364

Annales Universitatis Paedagogicae Cracoviensis

Studia ad Didacticam Mathematicae Pertinentia 14(2022)

ISSN 2080-9751
DOI 10.24917/20809751.14.2

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The mixtilinear excircle vs the mixtilinear incircle*

Abstract. Many theorems concerning incircle of a random triangle can be transferred by analogy onto it's excircle. In the following paper we aim to show analogies between mixtilinear incircle and mixtilinear excircle by presenting variants of theorems proved in (Pater, Sochacki, 2020).

1. Introduction

Leon Bankoff divided triangles in the Euclidean space into rectilinear, mixtilinear or curvilinear depending on whether all, some or none of the bounding lines are straight (Bankoff, 1983). He derived a trigonometric formula for radius of the circle tangent to two sides of a triangle and an arc of its circumcircle internally. Hence the term mixtilinear incircle. However, it's not the first time mathematicians became aware of these, since mixtilinear circles date back to 19th century Japan, where they were the main characters of a few San Gaku riddles.

Paul Yiu proved in 1999 via barycentric coordinates method what is written below as theorem 12 for mixtilinear incircles (Yiu, 1999), which was further generalized by Stanley Rabinowitz, where he considered 'pseudo-incircles' instead of mixtilinear incircles (Rabinowitz, 2006). Later on, Yiu listed the barycentric coordinates of points, lines and Apollonian circles associated with both mixtilinear incircles and excircles (Yiu, 2023). In 2006 Nguyen and Salazar gave additional insights on radical axes and radical centers of mixtilinear circles (Nguyen, Sochacki, 2006).

Every non-degenerate triangle ABC has exactly three mixtilinear excircles and incircles we denote respectively as $\omega_A, \omega_B, \omega_C$ and π_A, π_B, π_C. ω_A, π_A are defined as circles inscribed in the internal angle BAC tangent externally/internally to the circumcircle of triangle ABC. Analogously we define the remaining circles. In the following paper we consider a random triangle ABC and it's associated points:

*2020 Mathematics Subject Classification: *51M04, 51N20*

Keywords and phrases: *mixtilinear circle, mixtilinear excircle, mixtilinear incircle*

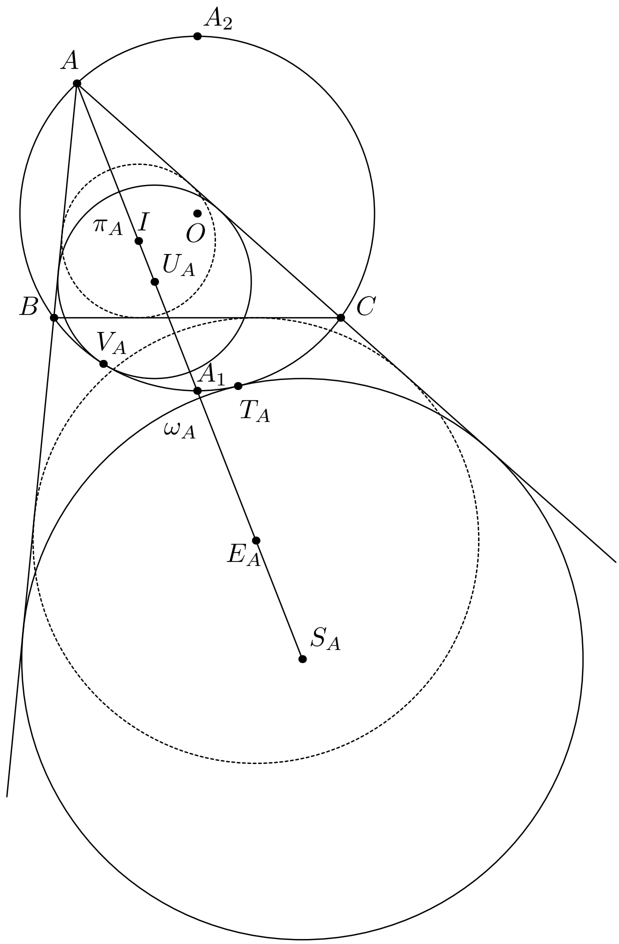

Figure 1. Labels.

- I - incenter,
- O - circumcenter,
- A_1 - center of arc BC opposite to point A,
- A_2 - center of arc BC containing point A,
- E_A - center of excircle tangent to side BC,
- S_A - center of ω_A,
- T_A - touchpoint of ω_A and circumcircle ABC,
- U_A - center of π_A,
- V_A - touchpoint of π_A and circumcircle ABC.

Similarly, we define points B_1, B_2, C_1, C_2 and points E, S, T, U, V with subscripts B or C.
Oriented and non-oriented angles are noted respectively with symbols $\measuredangle, \angle$. Finally, we adopted the notation $XY^{\rightarrow}$ as the ray with initial point X passing through Y.

2. Mixtilinear excircle on cartesian plane

Lemma 1. *Let a circle ω lie on the plane at the same side of line AB as point C. If ω is externally tangent to circumcircle ABC at X and line AB at Y, there holds $C_2 \in YX^{\rightarrow}$ and*

$$C_2X \cdot C_2Y = C_2B^2.$$

Proof. There exists homothethy θ centered at X transforming ω into circumcircle ABC. Tangent to circle ω is transformed by θ into tangent to circumcircle ABC parallel to line AB lying on the same side of line AB as X, therefore passing through C_2. Hence $\theta(Y) = C_2$ which concludes $C_2 \in YX^{\rightarrow}$.
Observe that $\measuredangle BC_2X = -\measuredangle YC_2B$ and $\measuredangle BXC_2 = \measuredangle BAC_2 = \measuredangle C_2BA = -\measuredangle YBC_2$ imply $\triangle C_2BX \sim \triangle C_2YB$ in the opposite orientation and $C_2X \cdot C_2Y = C_2B^2$. $\square$

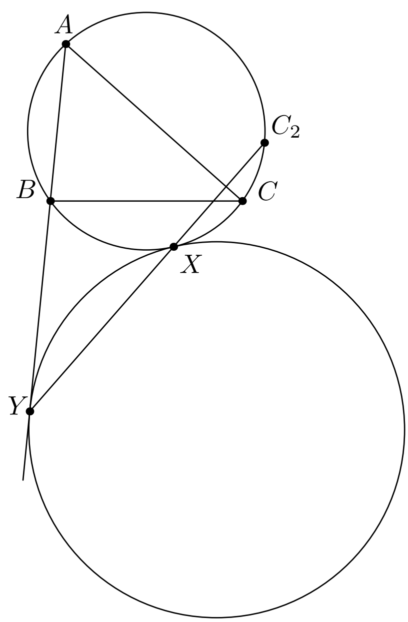

Figure 2. Lemma 1.

Theorem 1. *If P, Q are points of tangency of ω_A and lines AB, AC respectively, then $C_2 \in PT_A^{\rightarrow}$ and $B_2 \in QT_A^{\rightarrow}$.*
Mixtilinear incircle version: *If P, Q are points of tangency of π_A and lines AB, AC respectively, then $C_1 \in V_AP^{\rightarrow}$ and $C_1 \in V_AQ^{\rightarrow}$. (see theorem 1.4 in (Pater, Sochacki, 2020))*

Proof. The first property is direct conclusion from the lemma. The second is analogous property when swapping vertices B and C. □

Theorem 2. *(If $P \in AB$ and $Q \in AC$ are points of tangency of ω_A with extensions of triangle ABC sides, then E_A is the midpoint of segment PQ.*
Mixtilinear incircle version: *If $P \in AB$ and $Q \in AC$ are points of tangency of π_A with sides of triangle ABC, then I is the midpoint of segment PQ. (see theorem 1.1 in (Pater, Sochacki, 2020))*

Proof. By lemma we conclude that $\{P\} = AB \cap T_AC_2$ and $\{Q\} = AC \cap B_2T_A$. Lines BB_2, CC_2 are external angle divisors of triangle ABC and intersect at E_A. Considering Pascal's theorem in hexagon $ABB_2T_AC_2C$ we get collinearity of points P, E_A, Q. Because ω_A is inscribed in internal angle BAC, line AE_A is the bisector of segment PQ, therefore E_A is the midpoint of segment PQ. □

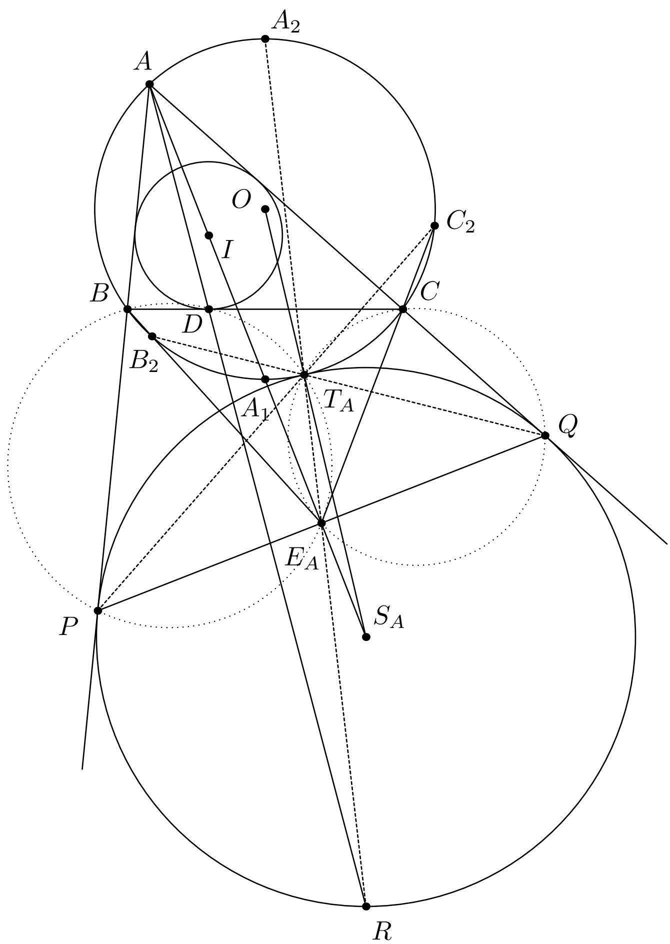

Figure 3. Theorems 1.–5.

Lemma 2. *Quadrilateral $A_2B_2E_AC_2$ is a parallelogram.*

Proof. Notice $\measuredangle IBE_A = 90° = \measuredangle ICE_A$, hence quadrilateral IBE_AC is cyclic. Moreover $\angle A_2BC = \angle A_2CB = 90° - \frac{1}{2}\angle BAC = \angle BE_AC$. Lines A_2B, A_2C are tangent to circumcircle BCE_A and line E_AA_2 is a symmedian of the triangle BCE_A. From $\measuredangle BB_2A_2 = \measuredangle BCA_2 = \measuredangle BE_AC$ we derive $A_2B_2 \parallel C_2E_A$. Analogously we prove $A_2C_2 \parallel B_2E_A$. □

Theorem 3. *Points T_A, A_2 lie on E_A-symedian of triangle BCE_A.*
Mixtilinear incircle version: *Points V_A, A_2 lie on I-symedian of triangle BIC. (see theorem 1.2 in (Pater, Sochacki, 2020))*

Proof. From lemma 2 we deduce midpoint of segment B_2C_2 lies on segment A_2E_A. Consider homothethy θ transforming ω_A into circumcircle ABC. By lemma 1 $\theta(P) = C_2, \theta(Q) = B_2$. Theorem 1 yields that E_A is transformed by θ into midpoint of B_2C_2. Therefore $T_A \in E_AA_2$. □

Corollary 1. *Point T_A lies on circle with diameter A_1E_A.*

Theorem 4. *Let $R \neq T_A$ be the intersection of line E_AT_A with ω_A. Then $PR \parallel BE_A, QR \parallel CE_A$ and $S_AR \perp BC$.*
Mixtilinear incircle version: *Let $R \neq T_A$ be the intersection of line IV_A with π_A. Then $PR \parallel BI, QR \parallel CI$ and $U_AR \perp BC$. (see corollary 1.9.1 in (Pater, Sochacki, 2020))*

Proof. Consider homothety θ centered at T_A transforming ω_A into circumcircle ABC. By lemma 1 and theorem 3 we have $\theta(P) = C_2, \theta(Q) = B_2, \theta(R) = A_2, \theta(S_A) = O$. Hence $PR \parallel C_2A_2$. Lemma 2 yields $C_2A_2 \parallel B_2E_A$, therefore $PR \parallel BE_A$. Analogously we prove $QR \parallel CE_A$. Moreover $S_AR \parallel A_2O$ and $A_2O \perp BC$ ends the proof.

Corollary 2. *If D is the point of tangency of incircle with side BC, then points A, D, R are collinear (homothety centered at A).*
Mixtilinear incircle version: *If D is the point of tangency of A-excircle with side BC, then points A, D, R are collinear. (see theorem 2.17 in (Pater, Sochacki, 2020))*

Theorem 5. *If $P \in AB$ and $Q \in AC$ are points of tangency of ω_A with extensions of triangle ABC sides then quadruples of points $(B, P, T_A, E_A), (C, Q, T_A, E_A)$ are concyclic.*
Mixtilinear incircle version: *If $P \in AB$ and $Q \in AC$ are points of tangency of π_A with sides of triangle ABC then quadruples of points $(B, P, V_A, I), (C, Q, V_A, I)$ are concyclic. (see theorem 1.3 in (Pater, Sochacki, 2020))*

Proof. Observe $\angle C_2BE_A = 90° - \frac{1}{2}\angle BAC = \angle BE_AC_2$ gives $C_2B = C_2E_A$. From lemma 1 we infer $C_2 \in \overrightarrow{PT_A}$ and $C_2P \cdot C_2T_A = C_2B^2 = C_2E_A^2$. Hence $\triangle C_2T_AE_A \sim \triangle C_2E_AP$ in the opposite orientation and $\measuredangle CE_AT_A = \measuredangle C_2E_AT_A = \measuredangle E_APC_2 = \measuredangle QPT_A = \measuredangle CQT_A$, which proves concyclicity of points (B, P, T_A, E_A). Proof for the second quadruple is analogous. □

Corollary 3. *Points T_A, C_2 lie on P-symmedian of triangle BPE_A.*
Mixtilinear incircle version: *Points V_A, C_1 lie on P-symmedian of triangle BPI.*

Proof. $\measuredangle E_A B C_2 = \measuredangle C_2 E_A B = \measuredangle E_A P B$. □

Theorem 6. *Let $P \in AB$ and $Q \in AC$ be the points of tangency of ω_A with extensions of triangle ABC sides. Let M, N be the intersections of line BC with lines PR, QR respectively. Then quadrilateral $MPQN$ is inscribed in circle centered at I.*
Mixtilinear incircle version: *Let $P \in AB$ and $Q \in AC$ be the points of tangency of π_A with sides of triangle ABC. Let M, N be the intersections of line BC with lines PR, QR respectively. Then quadrilateral $MPQN$ is inscribed in circle centered at E_A. (see theorem 1.10 in (Pater, Sochacki, 2020))*

Proof. From theorem 4 we infer $MP \parallel BE_A$, therefore $\angle PMB = \angle E_A BC = 90° - \frac{1}{2}\angle ABC = 90° - \frac{1}{2}\angle MBP$ which gives $MB = BP$. Therefore line BI is perpendicular to MP and $MI = PI$. Analogously we can prove $NI = QI$. Finally $AI \perp PQ$ gives $PI = QI$. □

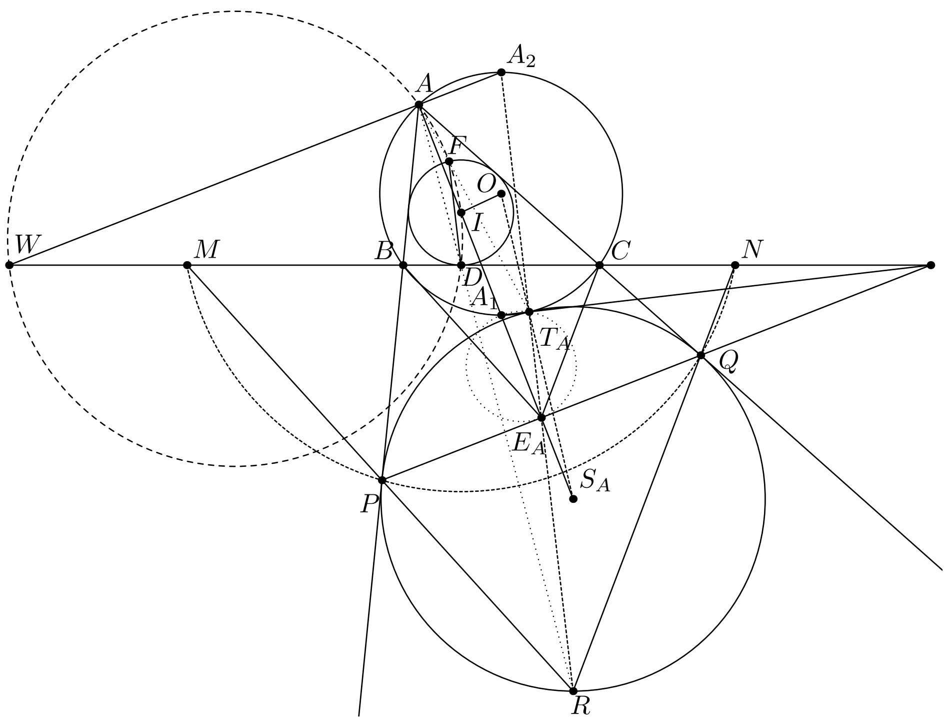

Figure 4. Theorems 6.–11.

Theorem 7. *Let $P \in AB$ and $Q \in AC$ be the points of tangency of ω_A with extensions of triangle ABC sides. Then lines $PQ, BC, T_A A_1$ are either concurrent or parallel.*
Mixtilinear incircle version: *Let $P \in AB$ and $Q \in AC$ be the points of tangency of π_A with sides of triangle ABC. Then lines $PQ, BC, V_A A_1$ are either concurrent or parallel. (see theorem 1.6 in (Pater, Sochacki, 2020))*

Proof. Lines AA_1, CC_2 intersect at E_A and by theorem 1 lines T_AA_1, BC intersect at P. Consider Pascal's theorem in hexagon $T_AA_1ABCC_2$: lines AB, T_AC_2, PE_A are either concurrent or parallel. □

We also give other proof.

Proof. Equality $\measuredangle IBE_A = 90° = \measuredangle E_ACI$ implies that the segment IE_A is a diameter of the circumcircle BCE_A. This fact conjoined with corollary 1 proves line PQ is the radical axis of circles with diameters IE_A, A_1E_A. By radical axis theorem applied to circumcircles $A_1T_AE_A$, BCE_A and ABC we have proved the required. □

Theorem 8. *Let D be the tangency of ABC incircle with side BC and F be the intersection of AT_A with incircle closer to the vertex A. Then $DF \parallel RT_A$.*

Proof. Consider homothethy θ centered at A transforming incircle into ω_A. Then $\theta(I) = S_A$ and line parallel to BC passing through I is transformed into line parallel to BC passing through S_A. Therefore theorem 4 yields $\theta(D) = R$ and segment DF is transformed by θ into segment RT_A giving $DF \parallel RT_A$. □

Theorem 9. *Pentagon $WAFID$ is inscribed in circle with diameter WI.*

Proof. Note perpendicular lines $AA_1 \perp WA_2$ and $WD \perp A_1A_2$. Hence $\measuredangle AWD = \measuredangle AA_1A_2$. Theorem 8 gives $\measuredangle AA_1A_2 = \measuredangle AT_AA_2 = \measuredangle AT_AR = \measuredangle AFD$. Therefore quadrilateral $WAFD$ is cyclic. Observe $\measuredangle WDI = 90° = \measuredangle WAI$, so points A, D lie on circle with diameter WI. □

Corollary 4. $\measuredangle BAD = \measuredangle BAI - \measuredangle DAI = \measuredangle IAC - \measuredangle IAF = \measuredangle T_AAC$

Theorem 10. *$\triangle BAD \sim \triangle T_AAC$ in the same orientation.*
Mixtilinear incircle version: *If E_A-centered excircle is tangent to the side BC at point G then $\triangle BAG \sim \triangle V_AAC$ in the same orientation. (see theorem 2.16 in (Pater, Sochacki, 2020))*

Proof. Note that inscribed angles equality $\measuredangle ABD = \measuredangle AT_AC$ with corollary above is equivalent to the desired similarity. □

Theorem 11. *Point I is the orthocenter of triangle WA_2E_A.*

Proof. By theorem 9 $\measuredangle WFI = \measuredangle WDI = 90°$ and $DI = FI$, hence $WI \perp DF$. Theorem 8 yields $WI \perp T_AR$. With theorem 3 we get $WI \perp A_2E_A$. Internal and external bisector of given angle are perpendicular, therefore $E_AI \perp WA_2$. □

Theorem 12. *Lines AT_A, BT_B, CT_C are concurrent in center of negative scale homothety transforming incircle into ABC circumcircle.*
Mixtilinear incircle version: *Lines AV_A, BV_B, CV_C are concurrent in center of positive scale homothethy transforming incircle into ABC circumcircle. (see theorem 2.19 in (Pater, Sochacki, 2020))*

Proof. Consider homothety θ_1 centered at A transforming incircle into ω_A with positive scale and homothethy θ_2 centered at T_A transforming ω_A into circumcircle ABC with negative scale. Then homothethy θ_3 centered at some point X on segment IO transforming incircle onto circumcircle ABC with negative scale is a composition of homotheties θ_1 and θ_2, therefore points A, X, T_A are collinear. Analogously we prove $X \in BT_B$ and $X \in CT_C$. □

3. Lengths

Throughout this section we assume a, b, c as the lengths of sides BC, CA, AB respectively.

Theorem 13. *Distances from T_A to vertices are given by formulas:*

$$AT_A = 2bc \cdot \sqrt{\frac{a}{(b+c-a)\,(a(a+b+c)-2(b-c)^2)}},$$

$$BT_A = c(a+b-c) \cdot \sqrt{\frac{a}{(b+c-a)\,(a(a+b+c)-2(b-c)^2)}},$$

$$CT_A = b(a+c-b) \cdot \sqrt{\frac{a}{(b+c-a)\,(a(a+b+c)-2(b-c)^2)}}.$$

Mixtilinear incircle version:

$$AV_A = 2bc \cdot \sqrt{\frac{a}{(a+b+c)\,(a(b+c-a)+2(b-c)^2)}},$$

$$BV_A = c(a+c-b) \cdot \sqrt{\frac{a}{(a+b+c)\,(a(b+c-a)+2(b-c)^2)}},$$

$$CV_A = b(a+b-c) \cdot \sqrt{\frac{a}{(a+b+c)\,(a(b+c-a)+2(b-c)^2)}}.$$

(see theorem 4.1 in (Pater, Sochacki, 2020))

Proof. By Stewart's theorem

$$AD^2 = \frac{AC^2 \cdot BD + AB^2 \cdot CD}{BC} - BD \cdot CD =$$

$$= \frac{c^2 \cdot \frac{1}{2}(a+b-c) + b^2 \cdot \frac{1}{2}(a+c-b)}{a} - \frac{(a+c-b)(a+b-c)}{4} =$$

$$= \frac{(b+c-a)\left(a(a+b+c)-2(b-c)^2\right)}{4a}.$$

Theorem 10 yields

$$AT_A = \frac{AB \cdot AC}{AD} = 2bc \cdot \sqrt{\frac{a}{(b+c-a)\,(a(a+b+c)-2(b-c)^2)}},$$

$$CT_A = \frac{AC}{AD} \cdot BD = b(a+c-b) \cdot \sqrt{\frac{a}{(b+c-a)\left(a(a+b+c)-2(b-c)^2\right)}}.$$

Analogously we get the formula for BT_A by swapping variables b and c in the formula for CT_A :

$$BT_A = c(a+b-c) \cdot \sqrt{\frac{a}{(b+c-a)\left(a(a+b+c)-2(b-c)^2\right)}}.$$

□

Theorem 14. *Let $P \in AB$ be the point of tangency of ω_A with extension of triangle ABC side. Radius length of ω_A is equal to*

$$PS_A = \frac{2bc}{b+c-a} \cdot \sqrt{\frac{(a+b-c)(a-b+c)}{(a+b+c)(b+c-a)}}$$

and

$$AS_A = \frac{4bc}{b+c-a} \cdot \sqrt{\frac{bc}{(a+b+c)(b+c-a)}}, \quad AP = \frac{2bc}{b+c-a}.$$

Mixtilinear incircle version: *Let $P \in AB$ be the point of tangency of π_A with side of triangle ABC. Radius length of π_A is equal to*

$$PU_A = \frac{2bc}{a+b+c} \cdot \sqrt{\frac{(a+b-c)(a-b+c)}{(a+b+c)(b+c-a)}}$$

and

$$AU_A = \frac{4bc}{a+b+c} \cdot \sqrt{\frac{bc}{(a+b+c)(b+c-a)}}, \quad AP = \frac{2bc}{a+b+c}.$$

(see corollary 1.1.1 and theorem 4.3 in (Pater, Sochacki, 2020))

Proof. Let Z be the point of tangency of A-excircle with line AB. Then $AZ = \frac{1}{2}(a+b+c)$ and $\triangle AZE_A \sim \triangle AE_AP$, so

$$AP = \frac{AE_A^2}{AZ}.$$

In the right triangle AZE_A we have

$$ZE_A = \frac{2[ABC]}{b+c-a} = \frac{\sqrt{(a+b+c)(a+b-c)(a-b+c)}}{2\sqrt{b+c-a}}$$

and

$$AE_A^2 = AZ^2 + ZE_A^2 = \frac{1}{4}(a+b+c)^2 + \frac{(a+b+c)(a+b-c)(a-b+c)}{4(b+c-a)} = bc \cdot \frac{a+b+c}{b+c-a}$$

therefore

$$AP = \frac{2bc}{b+c-a}.$$

Observe $ZE_A \parallel PS_A$ gives

$$AS_A = AP \cdot \frac{AE_A}{AZ} = \frac{AE_A^3}{AZ^2} = \frac{4bc\sqrt{bc}}{(b+c-a)\sqrt{(a+b+c)(b+c-a)}},$$

as well as

$$PS_A = ZE_A \cdot \frac{AP}{AZ} = \frac{2bc\sqrt{(a+b-c)(a-b+c)}}{(b+c-a)\sqrt{(a+b+c)(b+c-a)}}.$$

□

4. Coordinates

In the following section we set the coordinates on Argand plane as in theorem below and we treat each point as a complex number. More insights on this approach can be found in the book (Pater, Sochacki, 2020).

Theorem 15. *There exist complex numbers a, b, c such that $|a| = |b| = |c| = 1$ and*

$$A = a^2, B = b^2, C = c^2, A_1 = -bc, B_1 = -ca, C_1 = -ab.$$

(see theorem 2.13 in (Pater, Sochacki, 2020))

Corollary 5. *$I = -(ab + bc + ca)$ as the orthocenter of triangle $A_1B_1C_1$, $E_A = ab - bc + ca$ as the reflection of I with respect to point A_1 and $A_2 = bc$.*

Theorem 16.

$$T_A = a \cdot \frac{ab - 2bc + ca}{2a - b - c}$$

Mixtilinear incircle version:

$$V_A = -a \cdot \frac{ab + 2bc + ca}{2a + b + c}$$

(see theorem 2.14 in (Pater, Sochacki, 2020))

Proof. Since T_A lies on circumcircle ABC we have $T_A \cdot \overline{T_A} = |T_A|^2 = 1$. By theorem 3 points T_A, E_A, A_2 are collinear, hence

$$\frac{T_A - A_2}{E_A - A_2} = \overline{\left(\frac{T_A - A_2}{E_A - A_2}\right)},$$

$$\frac{T_A - bc}{ab - 2bc + ca} = \overline{\left(\frac{T_A - bc}{ab - 2bc + ca}\right)} = \frac{\frac{1}{T_A} - \frac{1}{bc}}{\frac{1}{ab} - \frac{2}{bc} + \frac{1}{ca}} = \frac{T_A - bc}{2a - b - c} \cdot \frac{a}{T_A},$$

$$T_A = a \cdot \frac{ab - 2bc + ca}{2a - b - c}.$$

□

Theorem 17. *Let $P \in AB$ and $Q \in AC$ be the points of tangency of ω_A with extensions of triangle ABC sides. Then*

$$P = \frac{a(ab + ca - 2bc) + 2b^2(c - a)}{c - b}, Q = \frac{a(ab + ca - 2bc) + 2c^2(b - a)}{b - c}.$$

Mixtilinear incircle version: *Let $P \in AB$ and $Q \in AC$ be the points of tangency of π_A with sides of triangle ABC. Then*

$$P = \frac{a(ab + ca + 2bc) + 2b^2(a + c)}{c - b}, Q = \frac{a(ab + ca + 2bc) + 2c^2(a + b)}{b - c}.$$

(see corollary 2.14 in (Pater, Sochacki, 2020))

Proof. From theorem 1 point P is the intersection of lines containing unit circle chords AB and T_AC_2. Therefore

$$P = \frac{A \cdot B \cdot (T_A + C_2) - T_A \cdot C_2 \cdot (A + B)}{A \cdot B - T_A \cdot C_2} =$$

$$= \frac{a^2b^2\left(a \cdot \frac{ab-2bc+ca}{2a-b-c} + ab\right) - ab \cdot a \cdot \frac{ab-2bc+ca}{2a-b-c}\left(a^2 + b^2\right)}{a^2b^2 - ab \cdot a \cdot \frac{ab-2bc+ca}{2a-b-c}} =$$

$$= \frac{a(ab + ca - 2bc) + 2b^2(c - a)}{c - b}.$$

Coordinates for Q are identical as for P except for swapping a and b. □

Theorem 18.

$$S_A = \left(\frac{ab - 2bc + ca}{b - c}\right)^2$$

Mixtilinear incircle version:

$$U_A = \left(\frac{ab + 2bc + ca}{b - c}\right)^2$$

(see theorem 2.14 in (Pater, Sochacki, 2020))

Proof. Points A, A_1, S are collinear, therefore:

$$\frac{S_A + bc}{a^2 + bc} = \overline{\left(\frac{S_A + bc}{a^2 + bc}\right)} = \frac{bc \cdot \overline{S_A} + 1}{a^2 + bc} \cdot a^2,$$

$$\overline{S_A} = \frac{S_A + bc - a^2}{a^2bc}$$

By theorem 2 P is the foot of S_A on line AB. Thus

$$2P = S_A + A + B - A \cdot B \cdot \overline{S_A},$$

$$2 \cdot \frac{a(ab + ca - 2bc) + 2b^2(c - a)}{c - b} = S_A + a^2 + b^2 - a^2b^2 \cdot \frac{S_A + bc - a^2}{a^2bc},$$

$$S_A = \frac{2ac(ab+ca-2bc)+4b^2c(c-a)-(c-b)\left((a^2+b^2)c-b(bc-a^2)\right)}{(c-b)^2},$$

$$S_A = \frac{(ab-2bc+ca)^2}{(c-b)^2}.$$

□

5. Acknowledgements

The authors are deeply grateful to the reviewers for valuable comments that contributed to the final version of this paper.

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